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Abstract

This study analyses the interactions between fertility dynamics and economic development in the overlapping generations model with human capital accumulation. We show that the economy develops with human capital accumulation, and hence, fertility rebound occurs only when the elasticity of substitution between consumption and the number of children is less than unity. As income increases, fertility increases due to income effect. When income is sufficiently large, households start investing educational investment for children, and hence the fertility decreases with human capital accumulation due to the substitution effect by educational investment for children. Finally, fertility increases again with income since the income effect is larger than substitution effect. Thus, this study clarifies the mechanism by which fertility rebound occurs in the economy.

JEL classifications: I25, J11, J13 Keywords: Human capital accumulation, Fertility rebound, Economic development

1. Introduction

As indicated by many studies and historical data, a shift from a positive relation-

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ship between income and fertility to a negative relationship occurs with economic development—that is, a demographic transition has been observed in developed countries. In addition, recent studies have found an increase in fertility, the so-called *fertility rebound*', in developed countries. For example, Luci-Greulich and Thévenon (2014) showed a shift from a negative relationship between per capita GDP and total fertility rate to a positive one in some developed countries, using OECD data from 1960 to 2007. Ohinata and Varvarigos (2020) found that the total fertility rate has increased in Scandinavian countries in recent decades. As indicated by Liao (2011), fertility dynamics is an important issue for economic development because it influences the growth path by increasing or decreasing population growth. This paper explains the mechanism of fertility rebound and analyses the interactions between fertility dynamics and economic development in a simple human capital accumulation model.

Many previous studies have attempted to explain the relationship between fertility dynamics and economic development. The seminal works by Becker and Lewis (1973), Barro and Becker (1989), de la Croix and Doepke (2003) and Yakita (2010) show a trade-off between child quality (educational investment for children) and child quantity (the number of children). Along the lines of unified growth theory, the seminal work by Galor and Weil (2000), Galor (2005a, b), and Becker et al. (2010) show the transition from Malthusian stagnation to modern growth, focusing on investment in human capital. Nakamura (2018) analysed the interactions between growth and demographic transition using the Stone-Geary utility function. While they show a demographic transition, they do not explain the fertility rebound observed in developed countries.

Fertility rebound has been explained in several theoretical studies. Day (2016) focuses on the role of workforce skill composition and external childcare. In Day (2016) model, individuals decide the number of children, choice of education,

and extent to which they use external childcare. As the number of skilled workers increases, population growth in the economy as whole decreases since fertility of the skilled is smaller than that of the unskilled. When there are enough skilled workers, population growth increases because people use external childcare. However, Day (2016) assumes indivisibilities in investment in human capital and the logarithmic utility function. In contrast, we consider continuous human capital accumulation and the constant elasticity of substitution (CES) utility function.

By incorporating external childcare into Galor and Weil (1997) model, Yakita (2018) shows the fertility rebound. When the female wage rate is sufficiently low, mothers do not work, so childcare services are not demanded and produced. When the female wage rate increases, she starts to work, and fertility decreases. When the wage is sufficiently large, households use external childcare, so fertility increases, i.e., the fertility rebound occurs. In contrast to their model, we consider a model with human capital and a CES-type utility function.

Futagami and Konishi (2019) constructed an overlapping generations model with endogenous mortality and R&D activities. In their model, physical costs (goods) of child-rearing and endogenous mortality play crucial roles in generating fertility rebound. They assume a positive relationship between life expectancy and wages. When wages are sufficiently small, fertility increases with wages due to the income effect being able to afford the child-rearing costs. As wages increase, longevity improves, and individuals increase their precautionary savings. Fertility with wages then decreases due to an increase in (precautionary) savings. When the wages are sufficiently high and longevity is very high, fertility increases again because the income effect of child-rearing costs is larger than the substitution effect of precautionary saving with an increase in longevity.

Assuming the intergenerational externalities of two paths for parental human

capital, Ohinata and Varvarigos (2020) explain the fertility rebound in the human capital accumulation model. In the intermediate stage of the economy, fertility increases and then decreases with human capital because of the trade-off between quality (educational investment) and quantity (the number of children). At a mature stage in the economy, parents can afford to invest in a desirable education for children without reducing the number of children. As a result, fertility in this stage again increases. Although they focus on human capital, they do not consider the role of external childcare and elasticity of substitution.

Focusing on the role of human capital accumulation and the trade-off between educational investment for children and fertility, this study explains the demographic transition and then fertility rebound. In particular, we construct the human capital accumulation model with a CES-type utility function, unlike previous studies. As indicated by Jones et al. (2011), the fertility-income relationship crucially depends on the elasticity of substitution between consumption and the number of children. When income increases, whether fertility increases or decreases depends on the income and substitution effects. However, previous studies do not consider the effects because they assume a logarithmic utility function; hence, they have not clarified the effects. One of the purposes of this study is to clarify the effects of human capital and elasticity of substitution.

We construct the overlapping generations model with human capital accumulation. This paper shows that fertility rebound occurs only when the elasticity of substitution is less than unity. At an early stage of economic development, fertility increases with income because of the income effect. As income increases, fertility decreases because of the substitution effect of human capital investment for children. Finally, fertility, as the economy grows, increases again since income effect is larger than substitute effect of educational investment

The remainder of this paper is organised as follows. Section 2 describes the

human capital accumulation model. Section 3 analyses the dynamics of the economy. Section 4 discusses the poverty trap. Section 5 concludes the paper.

2. The model

We construct an overlapping generations model with human capital accumulation. Consider the competitive equilibrium of an overlapping generations economy: each household consists of parents and children; each individual lives for two periods—childhood and adulthood. In the first period, she receives education, while she has children, works, and divides her income between consumption, child-rearing costs, and educational investment for children in adulthood.

2.1 Production and technology

For simplicity, we assume that production function is linear in labour.

$$Y_t = L_t, \tag{1}$$

where L_t is the total population in period *t*. We assume that the wage is always equal to one in the equilibrium in the labour market, that is, $w_t=1$ for all *t*.

2.2 Individuals

People gain utility from consumption c_i , the number of children n_i and educational investment for children e_i in period t. Hence, the preference of an individual of generation t is expressed by the following CES-type utility function.

$$u_{t} = \left\{ c_{t}^{\frac{\sigma-1}{\sigma}} + \gamma \left[\beta(\theta + e_{t})^{\frac{\sigma-1}{\sigma}} + n_{t}^{\frac{\sigma-1}{\sigma}} \right] \right\}^{\frac{\sigma}{\sigma-1}},$$
(2)

where $\sigma > 0$, $\beta \in (0, 1)$ and $\gamma \in (0, 1)$ represent the elasticity of substitution, the preference for educational investment, and the preference for children, respec-

⁽¹⁾ With $\sigma \rightarrow 1$, the utility function becomes the Cobb-Douglas function. This type of utility function is used in Borck (2011).

tively. The θ is the parameter used to obtain e_t convex function (de la Croix and $\binom{2}{2}$). Individuals allocate their income between consumption, child-rearing costs, and educational expenditure for children. Thus, her budget constraint becomes:

$$h_t = c_t + \phi n_t h_t + e_t, \qquad 0 < \phi < 1. \tag{3}$$

where h_t and ϕ denote human capital, that is, labour income for one unit of time and childcare time. Individuals of generation *t* choose their own consumption, number of children, and educational expenditure for children. Substituting (3) into (2), we can solve the following utility maximisation problem:

$$\max_{n_t,e_t}\left\{\left(h_t-\phi n_t h_t-e_t\right)^{\frac{\sigma-1}{\sigma}}+\gamma\left[\beta\left(\theta+e_t\right)^{\frac{\sigma-1}{\sigma}}+n_t^{\frac{\sigma-1}{\sigma}}\right]\right\}^{\frac{\sigma}{\sigma-1}}.$$

From the first-order condition for maximisation, we have the optimal educational expenditure for children and the number of children:

$$e_t = \frac{h_t - \phi n_t h_t - \tilde{\gamma} \theta}{1 + \tilde{\gamma}} \tag{4}$$

$$n_{t} = \frac{\mu(h_{t} + \theta)}{\mu \phi h_{t} + (\phi h_{t})^{\sigma}} \tag{5}$$

where $\tilde{\gamma} \equiv (\gamma\beta)^{-\sigma}$ and $\mu \equiv \beta^{-\sigma}/1 + (\gamma\beta)^{-\sigma}$. From (4) and (5), the optimal educational investment per child x_t is:

$$x_{t} = \frac{e_{t}}{n_{t}} = \frac{(\phi h_{t})^{\sigma} [h_{t} - \tilde{\gamma}\theta - (1 + \tilde{\gamma})\mu\theta(\phi h_{t})^{1 - \sigma}]}{\mu(1 + \tilde{\gamma})(h_{t} + \theta)}.$$
 (6)

In addition, the human capital of an individual in adulthood in period t+1 is assumed as follows:

$$h_{t+1} = \varepsilon (\theta + x_t)^{\eta} h_t^{\delta}, \qquad \varepsilon, \, \theta, \, \eta, \, \delta > 0, \quad \eta + \delta < 1.$$

The existence of θ implies that the children's human capital is positive if parents do not invest in education (de la Croix and Doepke 2003; Galor and Weil 1997).

⁽²⁾ In other words, when the income is sufficiently large, individuals start to invest in education for their children. Otherwise, individuals do not invest in it, i.e., $e_t=0$.

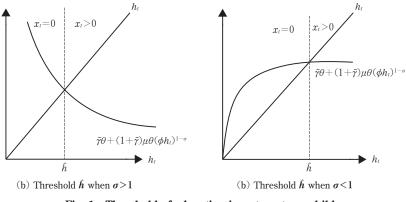


Fig. 1 Threshold of education investment per child

Since $\theta > 0$, human capital is positive even if parents do not invest in education, that is, $x_i=0$. h_i in Eq. (7) captures the intergenerational externalities of human capital accumulation. This reflects the educational effect acquired by watching from the parental background. Hence, human capital is accumulated by intergenerational externalities when $x_i=0$ (see de la Croix and Doepke 2003 and Fioroni 2010).

As illustrated in Fig. 1, \hat{h} represents the threshold of educational investment for children. If $h_t > \hat{h}$, then individuals invest in education for children, that is, x_t >0. Otherwise, $x_t=0$. It implies that household start investing education for children when income is sufficiently large. In this paper, we consider the trade-off between fertility and educational investment. Therefore, we obtain the optimal number of children and the educational investment per child as follows:

$$n_{t} = N(h_{t}) = \begin{cases} \frac{\gamma^{\sigma}h_{t}}{\gamma^{\sigma}\phi h_{t} + (\phi h_{t})^{\sigma}} & \text{if } h_{t} \leq \hat{h} \\ \frac{\mu(h_{t} + \theta)}{\mu\phi h_{t} + (\phi h_{t})^{\sigma}} & \text{if } h_{t} > \hat{h}, \end{cases}$$
(8)

(3) See de la Croix (2003).

$$x_{t} = \begin{cases} 0 & \text{if } h_{t} \leq \hat{h} \\ \frac{(\phi h_{t})^{\sigma} [h_{t} - \tilde{\gamma} \theta - (1 + \tilde{\gamma}) \mu \theta(\phi h_{t})^{1 - \sigma}]}{\mu(1 + \tilde{\gamma})(h_{t} + \theta)} & \text{if } h_{t} > \hat{h}, \end{cases}$$
(9)

If $h_t > \hat{h}$, then individuals start to invest in education.

3. Dynamics

In this section, we describe the dynamics of the economy and fertility dynamics. From (7) and (9), the dynamic system is expressed as follows:

$$h_{t+1} = F(h_t) = \begin{cases} \varepsilon \theta^{\eta} h_t^{\delta} & \text{if } h_t \leq \hat{h} \\ \varepsilon [\theta + x(h_t)]^{\eta} h_t^{\delta} & \text{if } h_t > \hat{h}, \end{cases}$$
(10)

When $h_i \leq \hat{h}$, individuals do not invest in education for children; hence, human capital is accumulated by intergenerational externalities. When income is sufficiently large, that is, $h_i > \hat{h}$, human capital is accumulated through educational investment and intergenerational transmission.

Now, we analyse the dynamic path of human capital. Differentiating both sides of Eq. (10) with respect to h_i , the following equation is obtained:

$$\frac{\partial F(h_{i})}{\partial h_{i}} = \begin{cases} \delta \varepsilon \theta^{\eta} h_{i}^{\delta-1} > 0 & \text{if } h_{i} \leq \hat{h} \\ \varepsilon \eta [\theta + x(h_{i})]^{\eta-1} \frac{\partial x(h_{i})}{\partial h_{i}} h_{i}^{\delta} + \varepsilon \delta [\theta + x(h_{i})]^{\eta} h_{i}^{\delta-1} > 0 & \text{if } h_{i} > \hat{h}, \end{cases}$$

$$(11)$$

where $\partial x(h_t) / \partial h_t > 0$. Hence, the human capital stock h_{t+1} monotonically increases with h_t .

We then examined the steady-state equilibria in the model. Since F(0) = 0, the economy falls into *a trivial* steady state, that is, $h_{t+1}=h_t=0$ in this scenario. Suppose the initial human capital $h_0>0$. As shown in Eq. (11), if $h_t \le \hat{h}$, $F'(h_t) > 0$ and $F''(h_t) = -(1-\delta)\delta\varepsilon\theta^{\eta}h_t^{\delta-2} < 0$. Thus, $F(h_t)$ is a concave function with respect to h_t for $h_t \le \hat{h}$. In addition, if $h_t > \hat{h}$, $F'(h_t) > 0$ and $\lim_{h\to\infty} h_{t+1}/h_t = \varepsilon [\phi^{\sigma}/\mu(1+\tilde{\gamma})]^{\eta}$

 $\infty^{\eta\sigma+\delta-1}$. Thus, if $\eta\sigma > 1-\delta$, then $F(h_t)$ is a convex function with respect to h_t for $h_t > \hat{h}$. On the other hand, if $\eta\sigma=1-\delta$, then $F(h_t)$ is a linear function for $h_t > \hat{h}$. Finally, $\eta\sigma < 1-\delta$, $F(h_t)$ is a concave function with respect to h_t for $h_t > \hat{h}$. To ensure that the economy converges to an asymptotically stable steady state, we impose the following assumption:

Assumption 1

$$\eta\sigma < 1-\delta$$

This study is interested in fertility dynamics with economic development and high steady-state equilibria. Hence, to ensure that the economy does not fall into the poverty trap, we impose the following assumption:

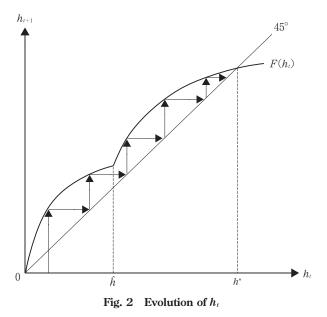
Assumption 2

$$\hat{h} \! < \! (arepsilon heta \eta)^{rac{1}{1-\delta}}$$

If $\hat{h} > (\varepsilon \theta \eta)^{1/1-\delta}$, then the economy falls into a poverty trap when the initial human capital h_0 is sufficiently low. As a result, we obtain the following proposition:

Proposition 1: Suppose that $\eta\sigma+\delta<1$ and $\hat{h}<(\varepsilon\theta\eta)^{1/1-\delta}$, then h_i monotonically increases, and the economy eventually converges to an asymptotically stable steady state h^* for any $h_0>0$.

Fig. 2 shows the evolution of human capital. When $h_0 < \hat{h}$, individuals do not invest in education for children, and the economy develops through intergenerational human capital transmission from parents to children. As the economy develops, an individual's income also increases. When $h_i > \hat{h}$, individuals start to invest in education for children, and the economy will then grow significantly. As a result, the economy approaches an asymptotically stable (high) steady-state h^*



for any $h_0 > 0$ as indicated by Proposition 1. This economic development path is

shown by de la Croix and Doepke (2003) and Fioroni (2010).

We then analyse fertility dynamics in the economy. The effect of income on the number of children becomes

$$\frac{\partial N(h_{l})}{\partial h_{t}} = \begin{cases}
\frac{(1-\sigma)(\gamma\phi h_{t})^{\sigma}}{\lceil (\phi h_{t})^{\sigma} + \gamma^{\sigma}\phi h_{t} \rceil^{2}} & \text{if } h_{t} \leq \hat{h} \\
\frac{\mu\phi^{\sigma}h_{t}^{\sigma-1}\lceil (1-\sigma)h_{t} - \theta\sigma - \mu\theta(\phi h_{t})^{1-\sigma} \rceil}{\lceil \mu\phi h_{t} + (\phi h_{t})^{\sigma} \rceil^{2}} & \text{if } h_{t} > \hat{h},
\end{cases}$$
(12)

The sign of $\partial N(h_i)/\partial h_i$ depends on the elasticity of substitution σ and the level of human capital h_i . When $\sigma = 1$ and $h_i \leq \hat{h}$, the number of children is constant. However, if $h_i > \hat{h}$, then the sign of $\partial n_i/\partial h_i$ is negative because of the substitution effect of educational investment. When $\sigma > 1$, fertility always decreases with human capital accumulation, that is, $\partial N(h_i)/\partial h_i < 0$. When $\sigma < 1$, fertility dynamics

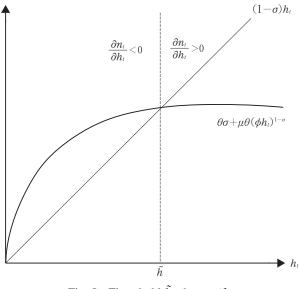


Fig. 3 Threshold \tilde{h} when $\sigma < 1$

depend on the level of human capital. If $h_t \leq \hat{h}$, the sign of $\partial N(h_t) / \partial h_t$ is always positive. However, if $h_t > \hat{h}$, the sign of $\partial n_t / \partial h_t$ depends on the income level. As Fig. 3 shows, when $h_t \leq \tilde{h}$, the sign of $\partial N(h_t) / \partial h_t$ is negative; hence, fertility decreases with income. On the other hand, when $h_t > \tilde{h}$, the sign of $\partial N(h_t) / \partial h_t$ is positive, and fertility increases with income. Hence, we obtain the following proposition.

Proposition 2: Suppose that $\sigma < 1$. Fertility dynamics depend on income level. Fertility increases with income for $h_t \leq \hat{h}$, and then decreases for $\hat{h} \leq h_t \leq \tilde{h}$. It increases again for $h_t \geq \tilde{h}$, hence, fertility rebound will occur.

$$\frac{\partial N(h_i)}{\partial h_i} \begin{cases} >0 & if \quad h_i < \hat{h} \\ <0 & if \quad \hat{h} < h_i < \tilde{h}, \\ >0 & if \quad h_i > \tilde{h}, \end{cases}$$
(13)

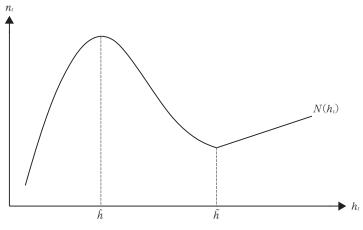


Fig. 4 Fertility dynamics when $\sigma < 1$

Fig. 4 illustrates the results of Proposition 2. When income is sufficiently low (i.e. $h_i < \hat{h}$), individuals do not invest in education for children. Hence, fertility increases with income because of the income effect of $\sigma < 1$. When $\hat{h} < h_i < \tilde{h}$, individuals invest in education for their children. Since the substitution effect of educational investment is larger than the income effect, fertility decreases for $\hat{h} < h_i$. However, when income increases relative to the price of educational investment for children (i.e., $h_i > \tilde{h}$, the substitution effect is smaller; hence, the income effect is larger than the effect. Thus, fertility increases with income, and fertility rebound occurs.

In this section, we demonstrate that not only educational investment but also the elasticity of substitution $\sigma < 1$ is a crucial role in fertility rebound. In the next section, we show that fertility rebound occurs regardless of the value of σ .

4. Discussion

In this section, we analyse the economy under the poverty trap. From (10) and (11), if $h_1^* \equiv (\epsilon \theta \eta)^{1/1-\delta}$. If $\hat{h} > h_1^* \equiv (\epsilon \theta \eta)^{1/1-\delta}$, the evolution of h_t is as shown in Fig. 120

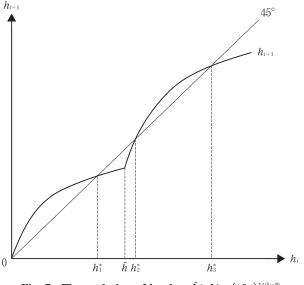


Fig. 5 The evolution of h_i when $\hat{h} > h_1^* \equiv (\varepsilon \theta \eta)^{1/(1-\delta)}$

5. As shown in Fig. 5, if the initial human capital h_0 is sufficiently low, the economy converges to an asymptotically stable low steady-state h_1^* and the economy falls into the poverty trap.

This situation is considered a *Malthusian regime* with low income and increased fertility. On the other hand, if the initial human capital h_0 is sufficiently high, the economy converges to an asymptotically stable high steady-state h_3^* . This situation is considered a *Modern growth regime* or *Fertility rebound regime*. When $\hat{h} < h_3^* < \tilde{h}$, i.e., human capital accumulation is sufficiently small, the economy is in *Modern growth regime*, where fertility decreased with income. In contrast, when $h_3^* > \tilde{h}$, i.e., human capital accumulation is sufficiently large, the economy is in *Fertility rebound regime*, where fertility increased with income.

One way to get out of poverty trap is to increase productivity in human capital production ε . From (10), we get the following:

$$\frac{\partial F(h_t)}{\partial \varepsilon} = \begin{cases} \theta^{\eta} h_t^{\delta} > 0 & \text{if } h_t \leq \hat{h} \\ [\theta + x(h_t)]^{\eta} h_t^{\delta} > 0 & \text{if } h_t > \hat{h}, \end{cases}$$
(14)

As indicated by (14), an increase in productivity shifts $F(h_t)$ upward. It implies that an increase in productivity promotes human capital accumulation for all h_t . We assume exogenous productivity in this paper. Endogenizing it should provide more interesting implication with respect to the relationship between economic development and human capital accumulation.

5. Conclusions

As indicated by many studies and historical data, a demographic transition has been observed in developed countries. In addition, recent studies have found an increase in fertility, the so-called 'fertility rebound', in developed countries. This study explains the mechanism which fertility rebound occurs in the economy constructing the overlapping generations model with human capital, and analyses the interactions between fertility dynamics and economic development. We show that fertility dynamics depend on the elasticity of substitution between consumption and the number of children. When elasticity is larger than unity, the fertility always decreases with income due to substation effects. However, when elasticity is smaller than unity, fertility increases with income due to the income effect. As income increases, individuals start to invest in education for children, and then fertility decreases due to the substitution effect. As income increases relative to education costs, the substitution effect is smaller. As a result, fertility increases again because the income effect is larger than the substitution effect.

To explain the fertility rebound, we assumed exogenous productivity and no external childcare services, e.g., babysitter and nursery school in this paper. Future research should incorporate external childcare services and endogenizing productivity in human capital production with the model to derive implications for economic development.

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