Demographic Transition and Economic Development: The Role of Child Costs

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Demographic Transition and Economic Development: The Role of Child Costs

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Abstract

This study analyzes the interaction between demographic transition and economic development, focusing on two child costs: time and physical child-rearing costs. To analyze the interactions, we construct two overlapping generations model: the human capital accumulation model and physical capital accumulation model. The two child costs, in particular, the physical, play a crucial role in non-monotonous fertility dynamics since they generate an income effect. In both growth models, an increase in physical child costs decreases fertility, whereas it promotes economic development through the dilution effect. As an increase in physical child cost encourages the start of investing in human capital, it facilitates a more rapid timing of demographic transition in the human capital accumulation model, thus freeing the economy out of the development trap. In contrast, it slows down the timing in physical capital accumulation model because of an increase in the income effect.

JEL classifications: I25, J11, J13, O11

Keywords: Demographic transition, Economic development, Child costs, Overlapping generations model.

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1. Introduction

This study analyzes the interactions between demographic transition and economic development, focusing on two child costs: time and physical child-rearing costs. As indicated by many studies and historical data, a shift from a positive relationship between income and fertility to a negative relationship with economic development, that is, demographic transition, has been observed in developed countries. As Liao (2011) indicates, demographic transition is an important issue for economic development since it influences the growth path by increasing or decreasing population growth. The analysis is important and useful not only for clarifying the growth path in developed countries but also for considering their economy.

Many previous studies have attempted to explain the relationship between demographic transition and economic development. Early works by Becker (1960), Becker and Lewis (1973), Willis (1973), and Barro and Becker (1989) show the trade-off between child quality and quantity. As the economy develops, individuals choose high quality, that is, more educational investment and a low quantity, that is, fewer children, and thus fertility decreases with economic development. Cáceres-Delpiano (2006) and Fernihough (2017) provide empirical evidence of the quality–quantity trade-off. In line with the quality–quantity theory, de la Croix (2003) explain the relationship between inequality in human capital among individuals and economic growth using a differential fertility model.[1] However-

(1) Demographic variables such as fertility and life expectancy have a significant impact on economic growth. Yakita (2010) demonstrates sustainable development in endogenous fertility model, and Chacraborty (2004) and Fanti and Gori (2014) analyze the effects of adult life expectancy on economic growth. In addition, in the human capital accumulation model, the effects of child mortality on economic development is analyzed by Azazert (2006) and Fioroni (2010).

ever, the trade-off appears only at a mature stage of economic development; therefore, they do not consider the early stage of economic development. Along the lines of unified growth theory, the seminal works by Galor and Weil (1999, 2000), Galor (2005a, b), and Becker et al. (2010) show the transition from Malthusian stagnation to modern growth, focusing on investment in human capital.

Using Stone–Geary utility function, Jones (2001) and Nakamura (2018) analyze the interactions between growth and demographic transition. Assuming the income effect by time rearing cost and substitution effect by the elasticity of substitution between consumption and the number of children, Nakamura (2018) shows non-monotonous fertility dynamics in a simple physical capital accumulation model. Furthermore, many previous studies focus on mortality and life expectancy. Reduction in mortality decreases fertility and, therefore, leads to demographic transition, as shown in Strulik (2003), Soares (2005), Cervelani and Sunde (2011, 2015), and Strulik and Weisdorf (2014), among others. However, this explanation is not supported by studies such as those by Mateos-Planas (2002) and Doepke (2005). Murphy (2009) find that women's social advancement is important for demographic transition and economic development.[(2)] Furthermore, Galor and Weil (1996) indicate that a decreasing gender gap due to the expansion of women's education promotes demographic transition and economic development.

This paper aims to explain demographic transition and clarify the effects of child costs, in particular, physical child-rearing costs, on demographic transition and economic development. To analyze this effect, we construct two overlapping generations models: the human capital accumulation model and the physical capi-

(2) Guinnane (2011) explains historical fertility transition in Europe and North America.

tal accumulation model. In this study, the existence of physical child cost plays a crucial role in the appearance of non-monotonous fertility dynamics, that is, demographic transition, as it generates an income effect. In other words, if there is no physical child cost, non-monotonous dynamics does not appear in the economy. In the human capital accumulation model, demographic transition is derived from the income effect of physical child costs and the substitution effect by the substitution between the educational investment for children and the number of children. At an early stage of economic development, individuals choose no investment in human capital and high fertility because of low income; hence, the economy has no substitution effect. As there is no substitution effect, fertility increases with income due to the income effect of physical child costs in the early stage of the economy. As the economy develops, an individual's income increases. At a mature stage of economic development, individual stars invest in human capital for children; therefore, the economy has both substitution and income effects. As the substitution effect is dominant, fertility decreases with income. The demographic transition is a crucial factor in economic development. If the timing of the demographic transition sufficiently slows, that is, individuals need a larger income to start educational investment, then the economy falls into the development trap due to high fertility and lack of education.

Nakamura (2018) considered the income effect by the existence of a minimum quality of consumption using Stone–Geary preferences. By incorporating physical child costs into Nakamura (2018) instead of minimum consumption, we analyze the effects of physical child costs on the relationship between demographic transition and economic development in the physical capital accumulation model. At an early stage of the economy, in contrast to the human capital accumulation model, the economy has both an income effect by physical child cost and a substitution effect by the substitution between consumption and fertility in the physical accumulation model. However, fertility decreases with income as the income effect dominates the substitution effect at the mature stage of the economy. In addition, we show the effects of an increase in physical child costs on demographic transitions and economic development. Fertility from additional child-rearing costs increases economic development through the dilution effect in both growth models. However, the effects on fertility in equilibrium and the timing of the demographic transition as the source of the substitution effect is different in each model.

Focusing on the role of child costs, this study presents the relationships between demographic transition and economic development in a simple but useful overlapping generations framework. Two child costs, in particular, the physical child cost, are a crucial factor for the appearance of a demographic transition. Using numerical simulations, Mateos-Planas (2002) indicates that child costs and technological progress, rather than mortality decline, are the main factors for demographic transition in Europe.

The remainder of this paper is organized as follows. Section 2 presents the human capital accumulation model. Section 3 analyzes the dynamics of demographic transition and economic development, and Section 4 demonstrates the effects of an increase in physical children's costs in the human capital accumulation model. Section 5 presents the physical capital accumulation model. Section 6 analyzes the dynamics of demographic transition and economic development, and Section 7 shows the effects of physical child costs in the physical capital accumulation model. Finally, Section 8 concludes the paper.

2. Human capital accumulation model

In this section, we analyze the interactions between demographic transitions and economic development in the human capital accumulation model. Consider the competitive equilibrium of an overlapping generation economy. Each individual lives for two periods: childhood and adulthood. In the first period, she receives an education. In the second period, she has children, works, and divides her income between consumption, child-rearing costs, and educational investment for children.

2.1 Production and technology

For simplicity, we assume that production function is linear in labor:

$$Y_t = L_t, \tag{1}$$

where L_t is the total working population in period t. Hence, the wage always equals 1; that is, $w_t = 1$ for all t in the equilibrium in the labor market.

2.2 Individuals

The human capital of an individual in adulthood in period $t+1$ is assumed as follows:

$$h_{t+1} = \varepsilon(\theta + e_t)^{\eta} h_t^{\delta}, \qquad \varepsilon,\ \theta,\ \eta,\ \delta > 0, \quad \eta + \delta < 1, \tag{2}$$

where e_t is the educational investment per child, and h_t is the stock of human capital in period t. Since $\theta > 0$, human capital is positive, even if parents do not invest in education (i.e., $e_t = 0$).

People gain utility from consumption c_t, the number of children n_t, and human capital of their children h_{t+1}. Hence, the preferences of the individual in generation t is expressed by the following utility function:

$$u_t = (1-\gamma)\log c_t + \gamma \log n_t h_{t+1}, \qquad \gamma > 0. \tag{3}$$

Individuals allocate their income to consumption, child-rearing costs, and educational expenditure for children. In particular, we assume two child costs: $m + \phi h_t$, which is time child-rearing cost ϕh_t, and physical child-rearing cost m needed to care for children (Boldrin and Jones, 2002). Thus, the budget constraint becomes

$$h_t = c_t + \phi n_t h_t + e_t n_t + m n_t, \qquad 0 < \phi < 1,\ m > 0. \tag{4}$$

Individuals of generation t choose their own consumption c_t, number of children n_t and educational expenditure per child e_t. By substituting (2) into (3), we solve the following utility maximization problem:

$$\max_{c_t, n_t, e_t} (1-\gamma)\log c_t + \gamma \log n_t + \gamma \log[\varepsilon(\theta + e_t)^{\eta} h_t^{\delta}]$$
$$\text{subject to} \quad h_t = c_t + \phi n_t h_t + e_t n_t + m n_t.$$

From the first-order condition for maximization, we have the optimal educational expenditure and number of children

$$n_t = \begin{cases} \dfrac{\gamma h_t}{(\phi h_t + m)} & \text{if} \quad h_t \leq \hat{h} \\[2ex] \dfrac{(1-\eta)\gamma h_t}{(\phi h_t + m - \theta)} & \text{if} \quad h_t > \hat{h}, \end{cases} \tag{5.a}$$

$$e_t = \begin{cases} 0 & \text{if} \quad h_t \leq \hat{h} \\[2ex] \dfrac{\eta(\phi h_t + m) - \theta}{1-\eta} & \text{if} \quad h_t > \hat{h}, \end{cases} \tag{5.b}$$

where $\hat{h} \equiv \theta/\eta\phi - m/\phi$, which represents the threshold of education investment and the turning point for the demographic transition. In addition, the fertility dynamics with human capital accumulation become

$$\frac{\partial n_t}{\partial h_t} = \begin{cases} \dfrac{\gamma m}{(\phi h_t + m)^2} > 0 & \text{if} \quad h_t \leq \hat{h}, \\[2ex] -\dfrac{(1-\eta)\gamma(\theta - m)}{(\phi h_t + m - \theta)^2} < 0 & \text{if} \quad h_t > \hat{h}. \end{cases} \tag{6}$$

When $h_t > \hat{h}$, whether an increase in income increases or decreases the number of children depends on $m - \theta$. To focus on demographic transitions and economic development, we assume that $\theta > m$. This implies that the number of children decreases with income, since the substitution effect is larger than the income effect when $h_t > \hat{h}$.(3) As can be seen from (6), if $m = 0$, fertility is constant when $h_t \leq \hat{h}$, whereas it decreases with income when $h_t > \hat{h}$ since the income effect disappears. Hence, physical child-rearing costs m plays crucial role in demographic transition.

Proposition 1 Suppose that $\theta > m$. In the human capital accumulation model, fertility increases with income at an early stage of the economy, while it decreases with income at a mature stage of the economy since an individual starts to invest in education for children. If $m=0$, then the non-monotonous motion of fertility does not appear in the economy.

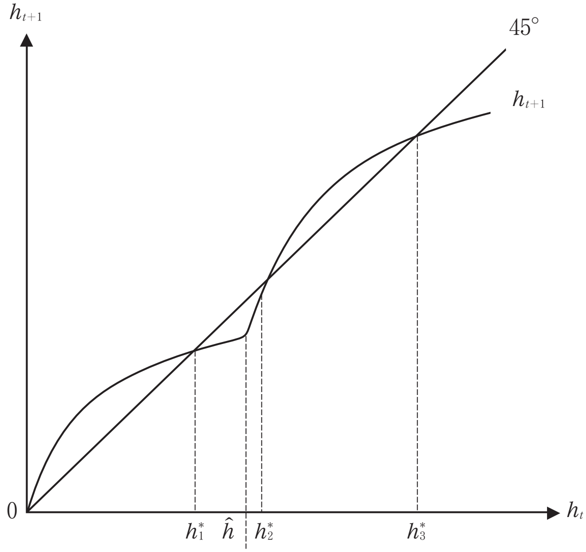
3. The dynamical system in the human capital accumulation model

In the human capital accumulation model, the dynamical system is expressed as follows:

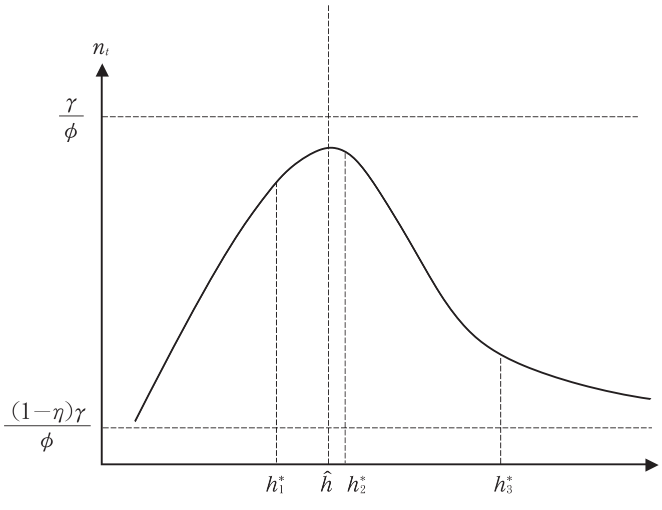
$$h_{t+1} = \begin{cases} \varepsilon \theta^{\eta} h_t^{\delta} & \text{if} \quad h_t \leq \hat{h} \\ \varepsilon \left[\dfrac{\eta(\phi h_t + m) - \theta}{1-\eta} \right]^{\eta} h_t^{\delta} & \text{if} \quad h_t > \hat{h}. \end{cases} \tag{7}$$

Fig. 1 shows the relationship between economic development and demographic transitions in the human capital accumulation model. When an economy is in the early stage of development, that is, $h_t \leq \hat{h}$, individuals do not invest in education for children because of low income. Hence, fertility increases with income owing to the income effect of the physical child cost. However, when an economy is sufficiently developed, that is, $h_t > \hat{h}$, as individuals start to invest in education for children, fertility decreases as income increases. Consequentially, the relationship between fertility and income shifts from positive to negative; that

(3) If $\theta < m$, an increase in income always raises the number of children since income effect is larger than substitute effect. In this case, the demographic transition does not emerge in the economy.



（a）Dynamical system of h_t



（b）Fertility dynamics of n_t

Fig. 1　Relationship between income and fertility

is, demographic transition occurs with economic development.(4)

When the initial value of human capital $h_0 = 0$, the economy converges to a trivial steady-state $h_{t+1}(0) = 0$. In addition, when $h_{t+1}(\hat{h}) \leq \hat{h}$, we can show that the steady-state $h_1^* = (\varepsilon\theta^{\eta})^{1/1-\delta}$. Hence, when $h_1^* \leq \hat{h}_t$, $h_{t+1}(\hat{h}) \leq \hat{h}$ holds. By contrast, when $h_1^* > \hat{h}_t$, $\hat{h}_{t+1}(\hat{h}) > \hat{h}$ and therefore the economy never converges to h_1^*. From (7), h_1^* and $\hat{h}$, we obtain the following proposition.

Proposition 2 When $h_0 > 0$ and $h_1^* > \hat{h}$, the dynamical system described by (7) has a unique steady state. By contrast, when $h_0 > 0$ and $h_1^* \leq \hat{h}$, the economy has multiple steady states; therefore, the economy may fall into a development trap.

Proof. First, we suppose that $h_0 > 0$ and $h_1^* > \hat{h}$. Since $h_{t+1}(\hat{h}) > \hat{h}$, the economy never converges to h_1^*. $\partial h_{t+1}/\partial h_t = \varepsilon\eta^2\phi D_t^{\eta-1} h_t^{\delta} + \delta\varepsilon D_t^{\eta} h_t^{\delta-1} > 0$, where $D_t \equiv (\eta\phi h_t + \eta m - \theta)/1-\eta$. Also, $\lim_{h_t \to \infty} h_{t+1}/h_t = \varepsilon(\eta\phi/1-\eta)^{\eta}\infty^{\eta+\delta-1}$. Since we assume that $\eta+\delta<1$, $\lim_{h_t \to \infty} h_{t+1}/h_t = 0$. This implies that $\partial^2 h_{t+1}/\partial h_t^2 < 0$. Hence, h_{t+1} is a concave function for h_t, and therefore, a unique and locally asymptotically stable steady state exists in the economy. Next, we suppose that $h_0 > 0$, and $h_1^* \leq \hat{h}$. Since $h_{t+1}(\hat{h}) \leq \hat{h}$, the economy shows a low steady-state h_1^*. If $\forall h_{t+1} < h_t$ for $h_t \in (\hat{h}, \infty)$, then unique and local asymptotically stable steady-state h_1^* exists in the economy. If $\exists h_{t+1} \geq h_t$ for $h_t \in (\hat{h}, \infty)$, two steady states $\{h_1^*, h_2^*\}$ exist in the economy, where h_2^* is local asymptotically unstable, or three steady states $\{h_1^*, h_2^*, h_3^*\}$, where h_3^* is locally asymptotically stable. Hence, the economy falls into a development trap when $h_{t+1}(\hat{h}) \leq \hat{h}$.

Proposition 2 implies that the economy converges to a higher steady state

(4) Similarly, Galor (2012) indicates that the demand for human capital is the main trigger for economic development and decreasing fertility.

when the threshold of the demographic transition $\hat{h}$ is sufficiently small. In contrast, when $\hat{h}$ is sufficiently large, the economy falls into a development trap and converges to a lower steady-state h_1^* with high fertility and low income, that is, Malthusian stagnation, or a higher steady-state h_3^* with low fertility and high income, as shown in Fig. 1. This figure illustrates an economy with multiple steady states: locally asymptotically stable low steady-state h_1^* and locally asymptotically stable high steady-state h_3^*. At an early stage of the economy, individuals do not invest in human capital for children because of their low income. In this economy, an increase in income has an income effect of child costs on fertility, while the substitution effect of educational investment does not exist; hence, fertility increases with economic development. When individuals do not invest in education and have high fertility due to low income, the economy converges to a low steady-state h_1^*, which is considered a Malthusian stagnation. At a sufficiently mature stage of the economy, individuals start to invest in human capital for children due to their higher income. As the substitution effect of investing in human capital is dominant, fertility decreases with income. As the economy develops, educational investment increases, while fertility decreases; therefore, the economy converges to a high steady-state h_3^*, characterized by low population growth and high income.

Whether the economy falls into development depends on the threshold value of demographic transition $\hat{h}$. When the threshold is sufficiently small, the economy develops with the accumulation of human capital through a quicker shift from increasing to decreasing population growth. However, when the threshold is sufficiently large, the economy may fall into a development trap since individuals do not invest in human capital because of high fertility and low income. As a result, economic development and demographic transition have a high degree of interdependence.

4. Effects of child costs in the human capital accumulation model

This section analyzes the effects of an increase in the physical child cost m on fertility, demographic transition, and economic development in the human capital accumulation model. First, we show the effects of an increase in child physical costs on economic development. When $h_t \leq \hat{h}$, the increase in the physical child cost has no effect on h_{t+1} since $\partial e_t / \partial m = 0$. In contrast, when $h_t > \hat{h}$, the increase in the physical child cost increases h_{t+1} since $\partial e_t / \partial m > 0$.

$$\frac{dh_{t+1}}{dm} = \begin{cases} 0 \\ \frac{\varepsilon\eta^2}{1-\eta}\left[\frac{\eta(\phi h + m - \theta)}{1-\eta}\right]^{\eta-1} h_t^{\delta} > 0 & \text{if} \quad h_t > \end{cases} \qquad (8)$$

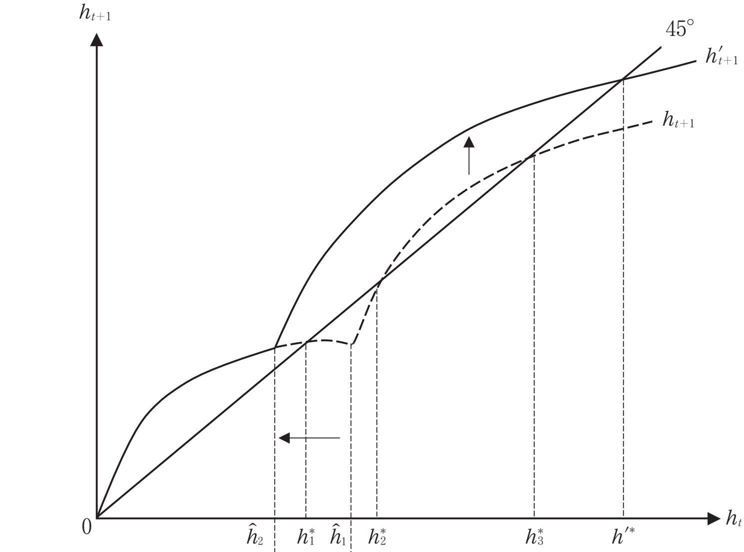
When $h_t > \hat{h}$, $dh^*/dm > 0$ since $dh_{t+1}/dm > 0$. Hence, an increase in physical child costs promotes economic development since it encourages educational investment for children.

Next, we show the effects on demographic transition and fertility in equilibrium. From (5) and $\hat{h} \equiv \theta/\eta\phi - m/\phi$, we obtain the following result:

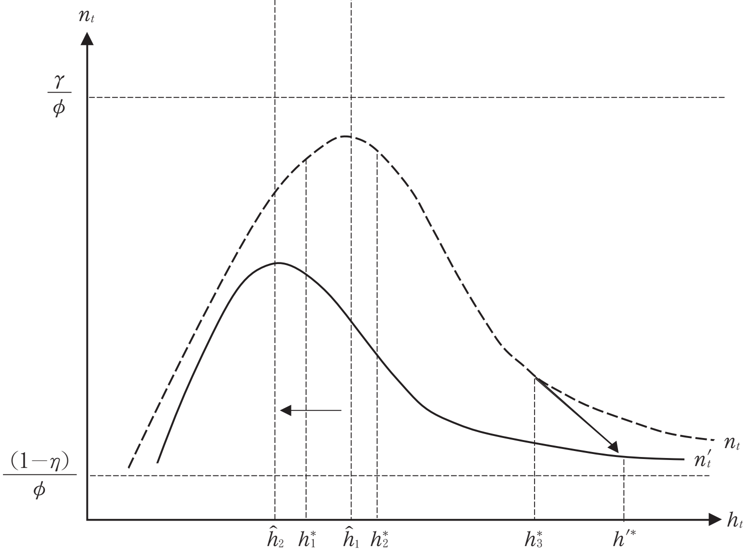
$$\frac{\partial \hat{h}}{\partial m} = -\frac{1}{\phi} < 0, \qquad \text{:and} \qquad (9)$$

$$\frac{dn^*}{dm} = \begin{cases} \underbrace{\frac{\partial n}{\partial m}}_{-} < 0 & \text{if} \quad h_t \leq \hat{h} \\ \underbrace{\frac{\partial n^*}{\partial m}}_{-} + \underbrace{\underbrace{\frac{\partial n^*}{\partial h^*}}_{-} \underbrace{\frac{\partial h^*}{\partial m}}_{+}}_{-} < 0 & \text{if} \quad h_t > \hat{h}. \end{cases} \qquad (10)$$

From (9) and (10), an increase in the physical child cost always decreases the threshold of demographic transition and fertility in equilibrium. An increase in physical child cost has two effects on fertility in equilibrium: direct effects through increasing child-rearing cost and indirect effects through substitution ef-



(a) Increase in the dynamical system of h_t



(b) Decrease in the fertility dynamics n_t

Fig. 2　Effects of increase in physical child cost

fect with increasing income. Consequently, we have the following proposition.

Proposition 3 An increase in physical child costs always decreases fertility in equilibrium and facilitates a more rapid timing of demographic transition. As they encourage investment in human capital, the economy develops with human capital accumulation. Hence, a sufficiently large increase in physical child cost leaves the economy out of the development trap.

Figure 2 illustrates the effects of an increase in physical child costs on economic development, fertility, and demographic transition. An increase in physical fertility increases human capital stock and greatly decreases fertility through two effects: direct and indirect. Hence, fertility greatly decreases, as shown in Fig. 2(b), and an increase in physical child cost promotes a shift from increasing to decreasing population growth. In other words, it shifts the threshold from $\hat{h}_1$ to $\hat{h}_2$. This intuition can be explained as follows: The physical cost of the child itself generates an income effect. However, an increase in the physical child cost increases the marginal cost of additional children, and therefore, individuals have a greatly reduced number of children. Due to this great decrease, individuals with low income can start to invest in human capital for children and, therefore, the economy develops without falling into a development trap. The economy eventually converges to h'^{*} with a high income and low fertility.(5) Therefore, in the human capital accumulation model, demographic transition and economic development depend crucially on physical child costs.

(5) Similar main results can be obtained with an increase in rearing child cost ϕ.

5. Physical capital accumulation model

In this section, we analyze the interactions between demographic transitions and economic development using the physical capital accumulation model. The model is based on incorporating physical child costs into Nakamura（2018）instead of the minimum quality of consumption. In doing so, we can show the role of child costs in the demographic transition and economic development. Consider the competitive equilibrium of an overlapping generation economy. Each individual lives in two periods: childhood and adulthood. All economic decisions are made in adulthood. Each individual determines consumption, bequests for children, and the number of children to maximize utility. In addition, bequests are used in the capital market of the economy.

5.1 Production and technology

We assume that the production function is characterized by a constant return-to-scale production function.

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}, \qquad A>0,\ 0<\alpha<1, \tag{11}$$

where K_t is physical capital and L_t is labor. Therefore, per-worker output becomes $y_t = Ak_t^{\alpha}$, where $k_t = K_t/L_t$ represents the per-worker stock of capital. We have the following in equilibrium under the assumption of full depreciation of capital.

$$1+r_t = \alpha Ak_t^{\alpha-1}, \qquad w_t = (1-\alpha)Ak_t^{\alpha}. \tag{12}$$

5.2 Individuals

Individuals gain from consumption c_t, bequest b_t and number of children n_t. Hence, we assume that the CES（Constant Elasticity of Substitution）type utility function is as follows:

$$u_t = \frac{(c_t^{\beta} b_t^{1-\beta})^{1-(1/\sigma)}}{1-(1/\sigma)} + \frac{n_t^{1-(1/\sigma)}}{1-(1/\sigma)} \qquad 0<\beta<1,\ \sigma>0, \tag{13}$$

where σ represents the elasticity of substitution between $c_t^{\beta} b_t^{1-\beta}$ and n_t. The individual allocates income to consumption, child-rearing costs, and bequests for children (see Nakamura, 2018). Similar to the human capital accumulation model in Section 2, we assume two child costs: the physical child-rearing cost and the time child-rearing cost needed to care for children. Thus, the budget constraint is given by

$$y_t = c_t + \phi n_t w_t + b_t + m n_t. \tag{14}$$

In adulthood, each individual decides on consumption c_t, bequest b_t, and number of children n_t. Similar to Nakamura (2018), we solve the utility maximization problem in two steps. In the first step, we solve the following utility maximization with the optimal number of children, as given:

$$\max_{c_t,\, b_t} c_t^{\beta} b_t^{1-\beta} \qquad \textit{subject to} \quad y_t = c_t + \phi n_t w_t + b_t + m n_t.$$

We obtain optimal consumption and bequest.

$$c_t = \beta (y_t - \phi n_t w_t - m n_t), \tag{15}$$

$$b_t = (1-\beta)(y_t - \phi n_t w_t - m n_t). \tag{16}$$

Substituting (15) and (16) into the utility function, we obtain the following utility function as a function of only n_t:

$$v(n_t) = \frac{\tilde{\beta}(y_t - \phi n_t w_t - m n_t)^{1-(1/\sigma)}}{1-(1/\sigma)} + \frac{n_t^{1-(1/\sigma)}}{1-(1/\sigma)'} \tag{17}$$

where $\tilde{\beta} = [\beta^{\beta}(1-\beta)^{1-\beta}]^{\frac{\sigma-1}{\sigma}}$. In the second step, we obtain the optimal number of children to maximize utility function (17):

$$\max_{n_t} \left\{ \frac{\tilde{\beta}(y_t - \phi n_t w_t - m n_t)^{1-(1/\sigma)}}{1-(1/\sigma)} + \frac{n_t^{1-(1/\sigma)}}{1-(1/\sigma)} \right\}.$$

The first-order condition is

$$n_t = (\tilde{\beta}\phi w_t + \tilde{\beta} m)^{-\sigma} [y_t - \phi n_t w_t - m n_t]. \tag{18}$$

From (18), we obtain optimal fertility:

$$n_t = \frac{w_t/(1-\alpha)}{(\phi w_t + m) + (\tilde{\beta}\phi w_t + \tilde{\beta} m)^{\sigma}}. \tag{19}$$

where $w_t/(1-\alpha) = y_t$.

If there is no physical child cost, $m=0$, we can rewrite (19) as follows:

$$n_t = \frac{w_t/(1-\alpha)}{\phi w_t + (\tilde{\beta}\phi w_t)^{\sigma}}. \tag{20}$$

When $m=0$, differentiating (20) with respect to w_t, we obtain

$$\frac{\partial n_t}{\partial w_t} = \frac{(\tilde{\beta}\phi w_t)^{\sigma}(1-\sigma)/(1-\alpha)}{[\phi w_t + (\tilde{\beta}\phi w_t)^{\sigma}]^2}, \tag{21}$$

and hence,

$$\frac{\partial n_t}{\partial w_t} \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr 1. \tag{22}$$

When $m=0$, the fertility behavior depends on the elasticity of substitution. If $\sigma < 1$, fertility increases with wages as the income effect is dominant. In contrast, if $\sigma > 1$, fertility decreases with wages since the substitution effect is dominant. Hence, if there is no physical child cost, monotonous fertility behavior is exhibited. In other words, non-monotonous behavior of fertility, that is, demographic transition, does not appear without physical child costs.

Next, we assume that $m > 0$. Differentiating (19) with respect to w_t, the fertility behavior is given as follows:

$$\frac{\partial n_t}{\partial w_t} = \frac{\{m[(\tilde{\beta}\phi w_t + \tilde{\beta} m)^{1-\sigma} + \tilde{\beta}] - (\sigma - 1)\tilde{\beta}\phi w_t\}(\tilde{\beta}\phi w_t + \tilde{\beta} m)^{\sigma-1}/(1-\alpha)}{[\phi w_t + m + (\tilde{\beta}\phi w_t + \tilde{\beta} m)^{\sigma}]^2}. \tag{23}$$

As shown in Fig. 3, the threshold or turning point of the demographic transition $\hat{w}$ exists when $\sigma > 1$. The relationships between n_t and w_t are expressed as

$$\frac{\partial n_t}{\partial w_t} \gtreqless 0 \Leftrightarrow w_t \lesseqgtr \hat{w}. \tag{24}$$

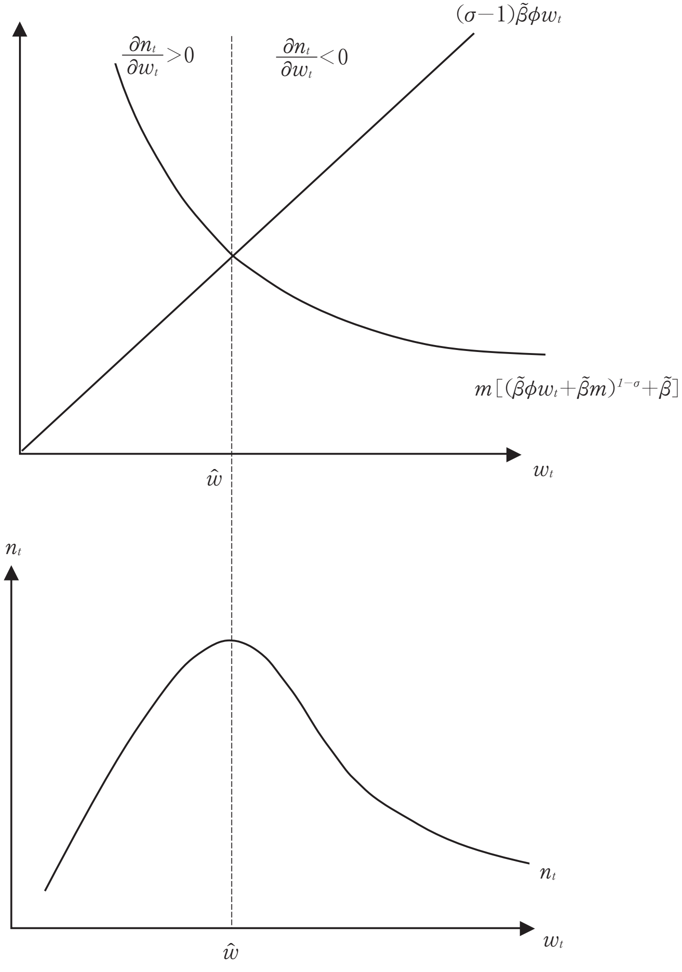


Fig. 3 Relationship between income and fertility

As illustrated in Fig. 3, when $w_t < \hat{w}$, that is, the economy is in the early stage of economic development, such as low income, fertility increases with wages, $\partial n_t / \partial w_t > 0$. In contrast, when $w_t > \hat{w}$, that is, the economy is in the mature stage of economic development, such as high wages, fertility decreases with wages, $\partial n_t /$

$\partial w_t < 0$. With physical child cost, the behavior of fertility is non-monotonous, that is, a demographic transition appears in the economy, as shown in Fig. 3. The existence of a physical child cost is a hurdle to rearing children and, therefore, generates an income effect. When income is very low, the marginal benefit of an additional child is very high, which prevents the substitution of consumption, bequests, and fertility. Even if $\sigma > 1$, when the economy is in the early stage, that is, wages are very low, the elasticity of substitution between consumption, bequest, and fertility is also very low due to the existence of physical child costs. Hence, when wages are sufficiently low, that is, $w_t < \hat{w}$ and fertility increases with wages as the income effect of physically rearing children dominates the substitution effect. As the economy develops and wages increase, the income effect becomes smaller since the hurdle by physical child cost is relatively lower. Thus, when wages are sufficiently high, that is, $w_t > \hat{w}$, fertility decreases with income since the substitution effect is larger than the income effect. As a result, the physical child cost generates non-monotonous fertility behavior with wages. These results are summarized in the following proposition.

Proposition 4　Without the physical child cost, fertility dynamics depend only on the elasticity of substitution; therefore, the monotonous motion of fertility appears in the economy. With physical child costs, fertility dynamics dramatically change with wages. When $\sigma > 1$ and wages are sufficiently low, fertility increases with wages since the income effect of physical child cost dominates the substitution effect. In contrast, when $\sigma > 1$ and wages are sufficiently high, fertility decreases with wages since the substitution effect is larger than the income effect. As a result, with physical capital cost and $\sigma > 1$, the non-monotonous behavior of fertility, that is, demographic transition, appears in the economy.

6. Dynamical system in physical capital accumulation model

In this section, we analyze the interactions between demographic transitions and economic development using the physical accumulation model. The inherited bequest is the capital stock in the current period, and the bequest to leave children is the capital stock in the next period. The aggregate capital stock in period $t+1$ becomes $K_{t+1}=L_t b_t$. Hence, the per-worker capital stock in period $t+1$ is $k_{t+1}=b_t/n_t$. Substituting (16) and (19) into $k_{t+1}=b_t/n_t$, the dynamical system is given by

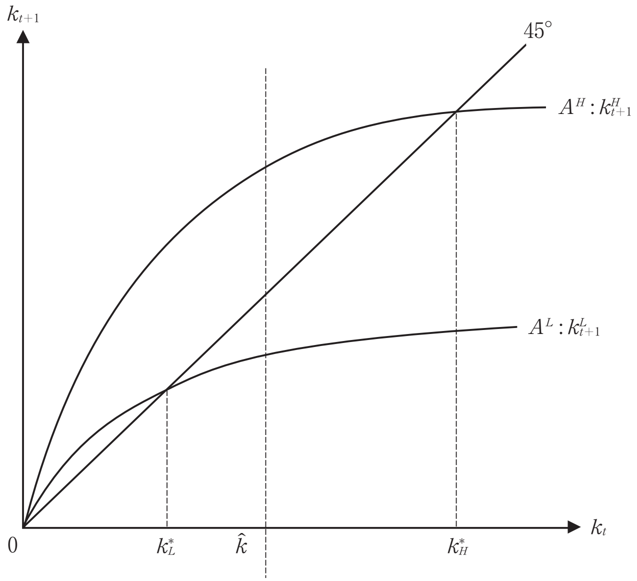
$$k_{t+1}=(1-\beta)(\tilde{\beta}\phi A k_t^{\alpha}+\tilde{\beta}m)^{\sigma}, \tag{25}$$

where $\partial k_{t+1}/\partial k_t=(1-\beta)\alpha\sigma\tilde{\beta}\phi A k_t^{\alpha-1}(\tilde{\beta}\phi A k_t^{\alpha}+\tilde{\beta}m)^{\sigma-1}>0$. In addition, $\lim_{k_t\to\infty} k_{t+1}/k_t =(1-\beta)(\tilde{\beta}\phi A)^{\sigma}\infty^{\alpha\sigma-1}$. When $\sigma<1/\alpha$, $\lim_{k_t\to\infty} k_{t+1}/k_t=0$, that is, $\partial^2 k_{t+1}/\partial k_t^2<0$ and k_{t+1} is a concave function for k_t; therefore, a unique locally asymptotically stable steady state exists in the economy. To ensure a stable steady state and analyze the demographic transition, we assume that $1<\sigma<1/\alpha$. This assumption implies that elasticity of substitution is not too large.

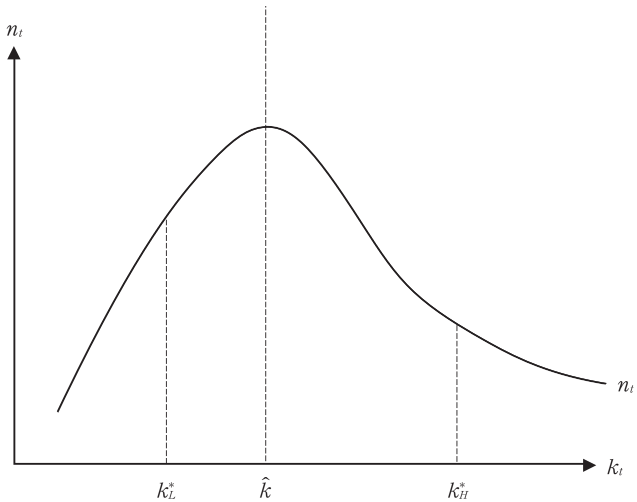
In addition, since $w_t=w(k_t)$, the following relationship holds for k_t and n_t.

$$\frac{\partial n_t}{\partial k_t}\gtreqless 0 \Leftrightarrow k_t \lesseqgtr \hat{k}. \tag{26}$$

Fig. 4 shows the relationship between economic development and the demographic transition. The relationship between fertility and per-worker capital changes from positive to negative. When technology is low, that is, $A=A_L$, the economy converges to k_L^*. Stable steady-state k_L^* is characterized by high fertility and low income. This situation is considered to be a Malthusian economy with low income and high fertility. When technology is high, that is, $A=A_H$, the economy converges to k_H^* with low fertility and high income. As the economy develops, fertility increases due to the income effect when income is low, that is, $k_t<\hat{k}$, and



（a）Dynamical system of k_t for different technology



（b）Fertility dynamics of n_t for different technology

Fig. 4　Relationship between income and fertility

it eventually decreases when k_t exceeds a certain level, that is, $k_t > \hat{k}$. Hence, the fertility dynamics become non-monotonous as the economy develops. Hence, we have the following proposition.

Proposition 5 Suppose that $1 < \sigma < 1/\alpha$. The economy converges to a locally asymptotically stable steady state. When technology is low, the economy converges to a steady state, with low income and high fertility. In contrast, when technology is high, fertility increases with income at an early stage of economic development and eventually decreases with income at a mature stage of economic development. Hence, the economy converges to a steady state with high income and low fertility.

7. Effects of child costs in physical capital accumulation model

In this section, we show the effects of an increase in the physical child cost m on fertility, demographic transition, and economic development in the physical capital accumulation model. First, we demonstrate the effects of k_{t+1}:

$$\frac{\partial k_{t+1}}{\partial m} = (1-\beta)\tilde{\beta}\sigma(\tilde{\beta}\phi A k_t^{\alpha} + \tilde{\beta}m)^{\sigma-1} > 0 \tag{27}$$

Hence, an increase in child physical costs encourages economic development. Next, we analyze the effects on fertility.

$$\frac{\partial n_t}{\partial m} = -\frac{w_t/(1-\alpha) + \tilde{\beta}\sigma w_t(\tilde{\beta}\phi w_t + \tilde{\beta}m)^{\sigma-1}/(1-\alpha)}{[\phi w_t + m + (\tilde{\beta}\phi w_t + \tilde{\beta}m)^{\sigma}]^2} < 0, \tag{28}$$

and hence

$$\frac{dn^*}{dm} = \begin{cases} \underbrace{\frac{\partial n^*}{\partial m}}_{-} + \underbrace{\underbrace{\frac{\partial n^*}{\partial w^*}}_{+}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial m}}_{+}}_{+} \gtrless 0 & \text{if} \quad k_t \leq \hat{k} \\ \underbrace{\frac{\partial n^*}{\partial m}}_{-} + \underbrace{\underbrace{\frac{\partial n^*}{\partial w^*}}_{-}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial m}}_{+}}_{-} < 0 & \text{if} \quad k_t > \hat{k}. \end{cases} \tag{29}$$

From (28), an increase in the physical child cost always decreases n_t. This decrease in fertility encourages physical capital accumulation per worker due to the dilution effect. In contrast, the effects on fertility in equilibrium n^* are ambiguous. An increase in the physical child cost on fertility in equilibrium has two effects: direct and indirect. The direct effect is the increase in the hurdle to rearing, and therefore, it decreases fertility. Conversely, the indirect effect is on fertility through increasing income. At an early stage of the economy, that is, $k_t \leq \hat{k}$, an increase in income increases fertility since the income effect is dominant, that is, the indirect effect is positive, while at the mature stage of the economy, that is, $k_t > \hat{k}$, it decreases since the substitution effect is dominant, that is, the indirect effect is negative. As a result, the effect of an increase in physical child cost on fertility in equilibrium is ambiguous when $k_t \leq \hat{k}$ due to negative and positive indirect effects. However, fertility always decreases in equilibrium when $k_t > \hat{k}$.

Finally, we show the effects of demographic transition. As can be seen from (23), the threshold of the demographic transition $\hat{k}$ depend on $g(w_t, m) = m[(\tilde{\beta}\phi w_t + \tilde{\beta}m)^{1-\sigma} + \tilde{\beta}]$ and $(\sigma - 1)\tilde{\beta}\phi w_t$. Hence, an increase in m shifts only $g(w_t, m)$. By differentiating $g(w_t, m)$ with respect to m, we can analyze the effects on the threshold $\hat{k}$.

$$\frac{\partial g(w_t, m)}{\partial m} = \tilde{\beta} + (\tilde{\beta}\phi w_t + \tilde{\beta}m)^{-\sigma}\{\tilde{\beta}m\sigma + \tilde{\beta}\phi w_t\} > 0. \tag{30}$$

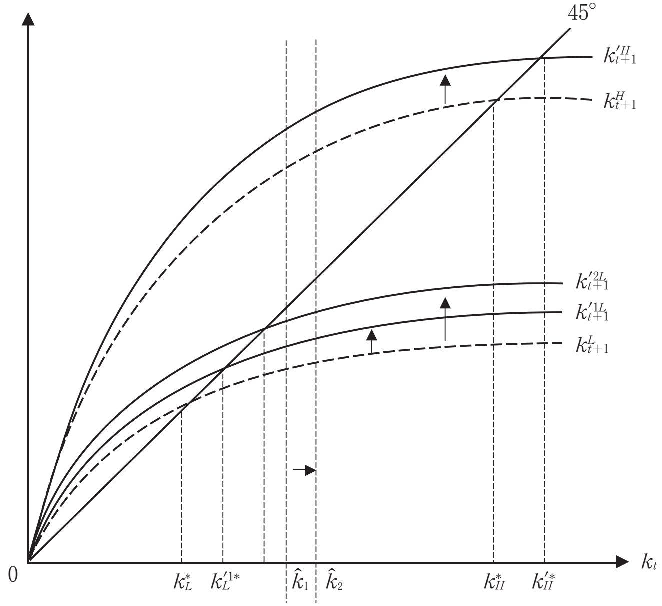
From (30), an increase in the physical child cost increases $g(w_t, m)$, and therefore, it increases the threshold $\hat{k}$. In other words, an increase in physical child costs slows down the timing of demographic transition:

$$\frac{\partial \hat{k}}{\partial m} > 0. \tag{31}$$

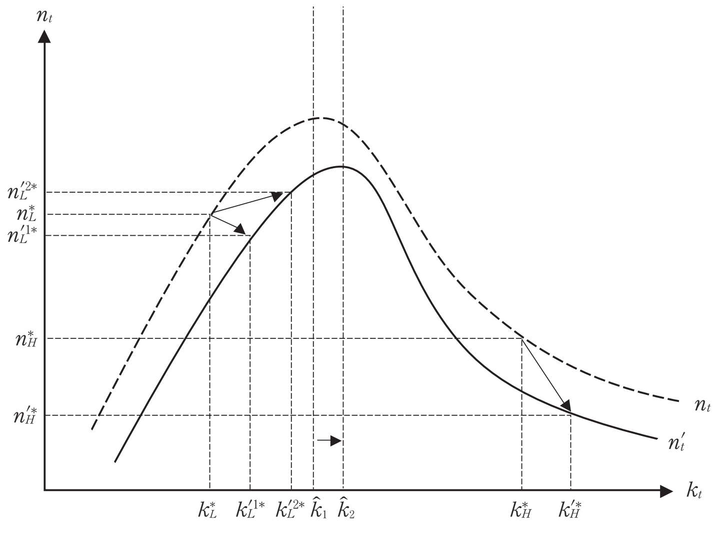
Unlike in the human capital accumulation model, an increase in physical child costs slows down the timing of demographic transition in the physical accumulation model. This intuition can be explained as follows. In the human capital accumulation model, the economy develops significantly by investing in human capital. However, individuals do not invest in human capital for children when income is low and fertility is high; therefore, fertility increases with income due to the income effect of physical child cost since there is no substitution effect at the early stage of economic development. When income is sufficiently large, that is, the economy is in the mature stage, individuals start to invest in human capital, and fertility decreases with income, since the substitution effect emerges. Hence, starting an educational investment generates a significant substitution effect in the human capital accumulation model. An increase in physical child costs promotes the timing of demographic transition since it encourages starting to invest in human capital, which generates a great substitution effect. In contrast, both income and substitution effects always exist from the early stage of the economy to the mature stage in the physical capital accumulation model. As an increase in physical child cost increases the hurdle of having children, it decreases fertility while facilitating a larger income effect. Assuming that the elasticity of substitution is not too large, individuals have more children because of the increasing income effect. This observation explains the slowdown in the timing of the demographic transition.

Proposition 6 An increase in physical child costs always decreases fertility, while it encourages per-worker capital stock. The effects of an increase in physical child costs on fertility at equilibrium depend on the stage of economic development. At the early stage of economic development, the effects are ambiguous, while it is always the negative at the mature stage of economic development. An increase in physical child costs slows down the timing of demographic transition, as it increases the income effect.

Fig. 5 illustrates the effects of an increase in physical child costs on economic development, fertility, and demographic transition. First, suppose that $A = A_L$, that is, $k_{t+1} = k^L_{t+1}$. An increase in physical child cost decreases fertility from n_t to n'_t. If its effects on k^L_{t+1} is smaller, then per-worker capital stock shifts from k^L_{t+1} to k'^{1L}_{t+1} due to the (smaller) dilution effect, while it decreases fertility in the equilibrium from n^*_L to n'^{1*}_L since the (negative) direct effect through increasing physical child cost is larger than the (positive) indirect effect through increasing income. Hence, the steady-state k'^{1*}_L or k'^{2*}_L is characterized by Malthusian stagnation with low income and high fertility. In contrast, if the effects of additional physical child cost on k^L_{t+1} is larger, then per-worker capital stock shifts from k^L_{t+1} to k'^{2L}_{t+1} due to the (larger) dilution effect, while it increases fertility in the equilibrium from n^*_L to n'^{2*}_L since the (positive) indirect effect is larger than the (negative) direct effect. Next, we suppose that $A = A_H$. Since the technology is high, that is, $k_{t+1} = k^H_{t+1}$, the economy eventually converges to k^*_H. An increase in the physical child cost increases per-worker capital stock from k^H_{t+1} to k'^H_{t+1} and therefore, the equilibrium shifts from k^*_H to k'^*_H. In addition, its effects on fertility in equilibrium decrease significantly from n^*_H to n'^*_H since both direct and indirect effects have negative effects on fertility. Hence, the economy converges to a steady-state k'^*_H with high income and low fertility. Finally, we show the effects of an increase in physical child costs on the demographic transition. As illustrated in Fig. 5, this increases the threshold from $\hat{k}_1$ to $\hat{k}_2$ as it encourages the income effect. Hence, it slows down the shift from increasing population to decreasing population growth, that is, the timing of demographic transition. Thus, similar to the human capital accumulation model, physical child cost plays a crucial role in economic development and demographic transition in the physical capital accumulation model.



(a) Increase in k_t for different technology



(b) Changes in n_t for different technology

Fig. 5 Effects of increase in physical child cost

8. Concluding remarks

This study analyzes the interactions between demographic transition and economic development in both the human and physical capital accumulation models, focusing on child costs. We assume that there are two types of child costs: physical and time child-rearing costs. In particular, the existence of physical child costs generates non-monotonous fertility dynamics, since it creates an income effect. Without physical child cost, non-monotonous fertility behavior does not appear in either growth model.

In both growth models, an increase in physical child costs decreases fertility due to an increase in rearing-child costs, while it promotes economic development through the dilution effect. However, its effects on fertility in equilibrium and demographic transition are different for the human capital accumulation model and physical capital accumulation model. The effects on fertility in equilibrium depend on the direct effect of increasing child-rearing costs, which decreases fertility, and indirect effects through increasing income, which depends on the stages of economic development. At an early stage of the economy, the effects are ambiguous in the physical capital accumulation model, since the direct effect is positive, and the indirect effect is positive. At the mature stage of the economy, fertility always decreases in equilibrium since both direct and indirect effects are negative. In contrast, regardless of the stage of economic development, fertility in equilibrium always decreases since the direct effect is always dominant. Consequently, an increase in the physical child cost always decreases fertility in equilibrium in human capital accumulation, while it is ambiguous in the physical capital accumulation model. As an increase in physical child cost encourages the initiation of investment in education, it facilitates a more rapid timing of demographic transition in the human capital accumulation model, and therefore the economy escapes the development trap. In contrast, an increase in physical child costs slows down the timing of demographic transition in the physical capital accumulation model since it increases the income effect. In fact, Mateos-Planas (2002) show that child costs are a crucial factor in the appearance of demographic transition.

Many studies discuss the determinants of demographic transition and interactions between demographic transition and economic development. Murtin (2013) shows that education is the determinant of demographic transition, while Lehl (2009) provids empirical evidence that the response of fertility to productivity depends on the stage of economic development. Mateos-Planas (2002) indicates that child costs and technological progress, rather than a decline in mortality, are crucial factors for the appearance of demographic transition. This study is consistent with his findings. In this study, two child costs—in particular, physical child-rearing costs—play crucial roles in demographic transition and economic development. If there is no physical child cost, non-monotonous fertility dynamics do not appear in either the human capital accumulation model or physical capital accumulation model. In contrast, we assume exogenous technology in this study. However, technological progress is an impossible factor for long-run growth, and it changes the population composition, as indicated by Nakamura (2018). Future research should endogenize technological progress to derive implications for demographic transition and economic development.

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