Capital-Skill Complementarity and Factor Intensity*

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Abstract

In the three-factor, two-goods models at constant prices, the impact of factor endowment changes on factor rewards depended only upon initial factor intensities. Technical complementarity in the individual production functions does not matter. We re-examine the role of capital-skill complementarity when determining the impacts of factor accumulation on factor rewards and intensities by using a two-step CES production function model. We show that the impact of factor accumulation on factor intensities is decomposed into two separate effects: the own-price substitution effect, and the complement effect. These effects depend not only on initial factor intensities, but also on technical complementarity.

1 Introduction

This paper examines the effects of factor accumulation on factor rewards and intensities, taking capital-skill complementarity into consideration.

There have been many attempts to analyze the impact of factor endowment changes on factor rewards using the generalized Hechscher-Ohlin model. For instance, Batra and Casas (1976), Ruffin (1981), and Thompson and Clark (1983) considered this issue by using three-factor, two-goods

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models at constant prices. In their models, the impact of factor endowment changes on factor rewards depended only upon initial factor intensities. Technical complementarity in the individual production functions does not matter.

Recently, capital-skill complementarity has been reconsidered in the context of inequality between unskilled and skilled labor (e. g., Stokey, 1996, Krusell et al., 2000). Therefore, we re-examine the role of capital-skill complementarity when determining the impacts of factor accumulation on factor rewards and intensities using a general equilibrium framework. We will show that whether the factors are technical complements or substitutes has important implications for factor movement.

2 The model

We built a two-sector model with three factors: unskilled labor, capital, and skilled labor. In order to highlight technical complementarity, we assume the following two-level CES (constant elasticity of substitution) production function.

$$X_j=(V_j^{-\delta_j}+L_j^{-\delta_j})^{-1/\delta_j},\quad j=1,2,\quad -1<\delta_j<\infty,\quad \delta_j\neq 0. \qquad (1)$$

$$V_j=(K_j^{-\rho_j}+S_j^{-\rho_j})^{-1/\rho_j},\quad j=1,2,\quad -1<\rho_j<\infty,\quad \rho_j\neq 0. \qquad (2)$$

X_j denotes the amount produced of the jth goods ($j=1, 2$), and L_j, K_j, and S_j denote respectively the amounts of unskilled labor, capital, and skilled labor used for production of the jth goods. V_j is a composite of K_j and S_j. As we will see later, unskilled labor is a substitute for both of the other inputs, while capital and skilled labor can be complements under this technology.

We assume a competitive, small, open economy with given world prices. Let P_j and W_i denote the price of goods j ($j=1, 2$) and the reward of factor

$i(i=L, K, S)$, respectively. The cost-minimizing amount of factor i used to produce one unit of goods $j\,(a_{ij})$ can be written as follows:

$$a_{Lj}=\left(\frac{P_j}{W_L}\right)^{1/(\delta_j+1)},\quad j=1,\ 2. \tag{3}$$

$$a_{Kj}=b_{Kj}b_{Vj},\quad b_{Kj}=\left(\frac{P_{Vj}}{W_k}\right)^{1/(\rho_j+1)},\quad b_{Vj}=\left(\frac{P_j}{P_{Vj}}\right)^{1/(\delta_j+1)},\quad j=1,\ 2. \tag{4}$$

$$a_{Sj}=b_{Sj}b_{Vj},\quad b_{Sj}=\left(\frac{P_{Vj}}{W_S}\right)^{1/(\rho_j+1)},\quad b_{Vj}=\left(\frac{P_j}{P_{Vj}}\right)^{1/(\delta_j+1)},\quad j=1,\ 2. \tag{5}$$

where

$$P_{Vj}=(W_K^{\rho_j/(\rho_j+1)}+W_S^{\rho_j/(\rho_j+1)})^{(\rho_j+1)/\rho_j},\quad j=1,\ 2. \tag{6}$$

Under perfect competition, the unit cost of each commodity equals its market price.

$$P_j=P_{Vj}b_{Vj}+W_La_{Lj},\quad j=1,\ 2. \tag{7}$$

Substituting (3) and (4) into the above condition and rewriting the result yields:

$$P_j=(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})^{(\delta_j+1)/\delta_j},\quad j=1,\ 2. \tag{8}$$

By using (8), we can eliminate P_{Vj} from (6).

$$(P_j^{\delta_j/(\delta_j+1)}-W_L^{\delta_j/(\delta_j+1)})^{(\delta_j+1)/\delta_j}=(W_K^{\rho_j/(\rho_j+1)}+W_S^{\rho_j/(\rho_j+1)})^{(\rho_j+1)/\rho_j},\quad j=1,\ 2. \tag{9}$$

Therefore, W_K and W_S can be written as a function of W_L, as follows:

$$W_K=W_K(W_L),\quad W_S=W_S(W_L). \tag{10}$$

Total differentiation yields:

$$\begin{pmatrix} b_{K1} & b_{S1} \\ b_{K2} & b_{S2} \end{pmatrix}\begin{pmatrix} dW_K \\ dW_S \end{pmatrix}=\begin{pmatrix} -(a_{L1}/b_{V1})dW_L \\ -(a_{L2}/b_{V2})dW_L \end{pmatrix} \tag{11}$$

We want to know the signs of dW_K/dW_L and dW_S/dW_L.

$$\frac{dW_K}{dW_L}=-\frac{B}{A} \tag{12}$$

$$\frac{dW_S}{dW_L}=-\frac{C}{A} \tag{13}$$

where

$$A=a_{K2}a_{S1}-a_{K1}a_{S2},\quad B=a_{L2}a_{S1}-a_{L1}a_{S2},\quad C=a_{L1}a_{K2}-a_{L2}a_{K1} \tag{14}$$

The signs of dW_K/dW_L and dW_S/dW_L depend on the order of a_{L1}/a_{L2}, a_{K1}/a_{K2}, and a_{S1}/a_{S2}. Let L, K, and S denote the supplies of unskilled labor, capital, and skilled labor, respectively. The market clearing conditions are as follows:

$$L=a_{L1}X_1+a_{L2}X_2 \tag{15}$$

$$K=a_{K1}X_1+a_{K2}X_2 \tag{16}$$

$$S=a_{S1}X_1+a_{S2}X_2 \tag{17}$$

We can obtain X_1 and X_2 from (16) and (17). Substituting them into (15) yields:

$$AL=BK+CS \tag{18}$$

As a_{ij} depends only on W_L, (18) determines W_L. If W_L is adjusted according to excess demand in unskilled labor markets, we have:

$$\dot{W}_L=\theta\left(\frac{B}{A}K+\frac{C}{A}S-L\right) \tag{19}$$

From the stability condition, we obtain:

$$D=(AB'-A'B)K+(AC'-A'C)S<0 \tag{20}$$

Taking total differentiation (18) results in:

$$\frac{dW_L}{dL}=\frac{A^2}{D}<0,\quad \frac{dW_L}{dK}=\frac{-AB}{D},\quad \frac{dW_L}{dS}=\frac{-AC}{D} \tag{21}$$

From (12), (13), and (21), we need to know the order of a_{i1}/a_{i2} $(i=L, K, S)$ to determine the impact of factor accumulation on rewards. For example, suppose that $a_{K1}/a_{K2}<a_{S1}/a_{S2}<a_{L1}/a_{L2}$. According to Ruffin's terminology, capital and unskilled labor are "extreme" factors and skilled labor is a "middle" factor. In this case, $A>0$, $B<0$, and $C>0$. From (12) and (13), $dW_K/dW_L>0$ and $dW_S/dW_L<0$, and from (21), $dW_L/dK<0$ and $dW_L/dS>0$. The results are summarized in Table 1, confirming Ruffin's theoretical findings that if the extreme factor increases, the reward of the other specific factor falls, whereas if the middle factor increases, the rewards of both of the other factors rise. For any given order of a_{i1}/a_{i2}, we can confirm Ruffin's results in our model.

Table 1: The impacts of factor accumulation

	W_L	W_K	W_S
$L\uparrow$	−	−	+
$K\uparrow$	−	−	+
$S\uparrow$	+	+	−

(In Table 1, it was assumed that $a_{K1}/a_{K2} < a_{S1}/a_{S2} < a_{L1}/a_{L2}$.)

3 The price elasticity of factor demand

Before analyzing the effects of factor accumulation on technical choice, it is useful to look at the price elasticity of factor demand.

From (5), (6), and (7) we obtain:

$$d\log a_{Sj} = \left(\frac{1}{\rho_j+1} - \frac{1}{\delta_j+1}\right) d\log P_{Vj} - \left(\frac{1}{\rho_j+1}\right) d\log W_S + \left(\frac{1}{\delta_j+1}\right) d\log P_j \tag{22}$$

$$d\log P_{Vj} = \left(\frac{W_K}{P_{Vj}}\right)^{\rho_j/(\rho_j+1)} d\log W_K + \left(\frac{W_S}{P_{Vj}}\right)^{\rho_j/(\rho_j+1)} d\log W_S \tag{23}$$

$$d\log P_j = \left(\frac{P_{Vj}}{P_j}\right)^{\delta_j/(\delta_j+1)} d\log P_{Vj} + \left(\frac{W_L}{P_j}\right)^{\delta_j/(\delta_j+1)} d\log W_L \tag{24}$$

Now, let us assume that W_S and P_j are constant. First, we calculate the effect of a change in W_K on a_{Sj}. From (22) and (23), we have:

$$\frac{\partial \log a_{Sj}}{\partial \log W_K} = \left(\frac{1}{\rho_j+1} - \frac{1}{\delta_j+1}\right)\left(\frac{W_K}{P_{Vj}}\right)^{\rho_j/(\rho_j+1)} \tag{25}$$

This means that if $\delta_j < \rho_i$, an increase in W_K brings about a reduction in a_{Sj}.

In this case, skilled labor and capital are complements.

Next, let us calculate the effect of a change in W_L on a_{Sj}. From (24), we have:

$$\frac{\partial \log P_{Vj}}{\partial \log W_L} = -\left(\frac{W_L}{P_{Vj}}\right)^{\delta_j/(\delta_j+1)} \tag{26}$$

Therefore, from (22), we have:

$$\frac{\partial \log a_{Sj}}{\partial \log W_L} = \left(\frac{1}{\delta_j+1} - \frac{1}{\rho_j+1}\right)\left(\frac{W_L}{P_{Vj}}\right)^{\delta_j/(\delta_j+1)} \tag{27}$$

If $\delta_j < \rho_j$, an increase in W_L raises a_{Sj}. The reason is as follows. Under constant world prices, an increase in W_L must bring about a decrease in P_{Vj}. As W_S is constant, an increase in W_L means there will be a decrease in W_K. When $\delta_j < \rho_j$—that is, when capital and skilled labor are complements—a decrease in W_K causes an increase in demand for skilled labor. We will call this the complement effect. On the other hand, when $\rho_j < \delta_j$, an increase in W_L decreases a_{Sj}. We call this the cross-price substitution effect.

Similarly, we can obtain:

$$\frac{\partial \log a_{Lj}}{\partial \log W_i} = \left(\frac{1}{\delta_j+1}\right)\left(\frac{P_{Vj}}{W_L}\right)^{\delta_j/(\delta_j+1)}\left(\frac{W_i}{P_{Vj}}\right)^{\rho_j/(\rho_j+1)}, \quad i=K, S. \tag{28}$$

As the sign of (28) is always positive, unskilled labor is a substitute for both of the other inputs.

4 Factor accumulation and factor intensities

In this section, we analyze the movement of factor intensities in response to an increase in skilled labor. From (3), (4), and (5) we can obtain:

$$\frac{d \log a_{Lj}}{d \log S} = -\left(\frac{1}{\delta_j+1}\right)\frac{\partial \log W_L}{\partial \log S} \tag{29}$$

$$\frac{d\log a_{ij}}{d\log S}=-\left(\frac{1}{\rho_j+1}\right)\frac{\partial \log W_i}{\partial \log S}$$

$$+\left(\frac{1}{\delta_j+1}-\frac{1}{\rho_j+1}\right)\left(\frac{W_L}{P_{Vj}}\right)^{\delta_j/(\delta_j+1)}\frac{\partial \log W_L}{\partial \log S},\quad i=K,S. \tag{30}$$

Equation (29) shows that the change in a_{Lj} is dominated by the own-price substitution effect. If W_L increases, a_{Lj} will decrease. As δ_j is smaller, the change is large. The second term of (30) is interpreted as a complement effect when $\delta_j<\rho_j$, or as a cross-price substitution effect when $\rho_j<\delta_j$. The change in a_{Kj} or a_{Sj} depends not only on the own-price substitution effect, but also on the complement effect caused by a change in W_L.

Consider what happens to the factor-intensity ranking when the supply of skilled labor increases. The change in a ratio of input coefficients is calculated as follows:

$$\frac{d\log(a_{L1}/a_{L2})}{d\log S}=\left(\frac{1}{\delta_2+1}-\frac{1}{\delta_1+1}\right)\frac{\partial \log W_L}{\partial \log S} \tag{31}$$

The change in a_{L1}/a_{L2} depends on which own-price substitution effects are bigger, those of industry 1 or those of industry 2.

$$\frac{d\log(a_{S1}/a_{S2})}{d\log S}=\left(\frac{1}{\rho_2+1}-\frac{1}{\rho_1+1}\right)\frac{\partial \log W_S}{\partial \log S}$$

$$+\left\{\left(\frac{1}{\delta_1+1}-\frac{1}{\rho_1+1}\right)\left(\frac{W_L}{P_{V1}}\right)^{\delta_1/(\delta_1+1)}\right.$$

$$\left.+\left(\frac{1}{\rho_2+1}-\frac{1}{\delta_2+1}\right)\left(\frac{W_L}{P_{V2}}\right)^{\delta_2/(\delta_2+1)}\right\}\frac{\partial \log W_L}{\partial \log S} \tag{32}$$

The change in a_{S1}/a_{S2} is more complicated. The sign of the first term is always the same because an increase in skilled labor causes a decrease in W_S. However, the sign of the second term depends on the technical complementarity and the sign of $\partial \log W_L/\partial \log S$. Assume that $a_{K1}/a_{K2}<a_{S1}/a_{S2}<a_{L1}/a_{L2}$ and that $\delta_1<\rho_2<\rho_2<\delta_2$. When S increases, W_S decreases and W_L

rises. Hence, a_{L1}/a_{L2} falls, a_{S1}/a_{S2} rises, and factor-intensity reversals occur.

5 Summary of results

Using a two-step CES production function model, we confirmed Ruffin's results. In addition, we showed that a change of factor intensity induced by factor accumulation is related to technical complementarity. We decomposed the technical change induced by factor accumulation into two separate effects: the own-price substitution effect, and the complement effect. The latter depends not only on initial factor intensities, but also on technical complementarity.

Appendix

Calculation of (3), (4) and (5)

A firm of industry j solves the following problem at the first stage.

$$\min \quad W_K K_j + W_S S_j$$
$$s.t. \quad V_j = (K_j^{-\rho_j} + S^{-\rho_j})^{-1/\rho_j}$$

where W_K, W_S, and V_j are given.

From the first order conditions, we obtain:

$$K_j = \left\{\left(\frac{W_S}{W_K}\right)^{\rho_j/(\rho_j+1)} + 1\right\}^{1/\rho_j} V_j$$
$$= \{W_K^{-\rho_j/(\rho_j+1)}(W_S^{\rho_j/(\rho_j+1)} + W_K^{\rho_j/(\rho_j+1)})\}^{1/\rho_j} V_j$$
$$= \left(\frac{P_{Vj}}{W_K}\right)^{1/(\rho_j+1)} V_j \tag{33}$$

Similarly, we obtain:

$$S_j = \left(\frac{P_{Vj}}{W_S}\right)^{1/(\rho_j+1)} V_j \tag{34}$$

The problem of the second stage is as follows:

$$\min \quad P_{Vj} V_j + W_L L_j$$
$$s.t. \quad X_j = (V_j^{-\delta_j} + L_j^{-\delta_j})^{-1/\delta_j}$$

where P_{Vj}, W_L, and X_j are given.

From the first order conditions, we obtain:

$$V_j=\left\{\left(\frac{P_{Vj}}{W_L}\right)^{-\delta_j/(\delta_j+1)}+1\right\}^{1/\delta_j}X_j$$

$$=\{P_{Vj}^{-\delta_j/(\delta_j+1)}(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})\}^{1/\delta_j}X_j \tag{35}$$

$$L_j=\left\{\left(\frac{P_{Vj}}{W_L}\right)^{\delta_j/(\delta_j+1)}+1\right\}^{1/\delta_j}X_j$$

$$=\{W_L^{-\delta_j/(\delta_j+1)}(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})\}^{1/\delta_j}X_j \tag{36}$$

Substituting (8) into (35) and (36) yields

$$V_j=\left(\frac{P_j}{P_{Vj}}\right)^{1/(\delta_j+1)}X_j \tag{37}$$

$$L_j=\left(\frac{P_j}{W_L}\right)^{1/(\delta_j+1)}X_j \tag{38}$$

From (38), we obtain:

$$a_{Lj}=\frac{L_j}{X_j}=\left(\frac{P_j}{W_L}\right)^{1/(\delta_j+1)} \tag{3}$$

From (33) ,(34) and (37), we obtain:

$$a_{Kj}=\frac{K_j}{X_j}=\left(\frac{P_{Vj}}{W_K}\right)^{1/(\rho_j+1)}\left(\frac{P_j}{P_{Vj}}\right)^{1/(\delta_j+1)} \tag{4}$$

$$a_{Sj}=\frac{S_j}{X_j}=\left(\frac{P_{Vj}}{W_S}\right)^{1/(\rho_j+1)}\left(\frac{P_j}{P_{Vj}}\right)^{1/(\delta_j+1)} \tag{5}$$

Calculation of (8)

The zero profit condition is written as follows:

$$P_j=P_{Vj}\left(\frac{V_j}{X_j}\right)+W_L\left(\frac{L_j}{X_j}\right) \tag{39}$$

Substituting (35) and (36) into the above condition yields (8).

$$P_j=P_{Vj}\{P_{Vj}^{-\delta_j/(\delta_j+1)}(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})\}^{1/\delta_j}$$

$$+W_L\{W_L^{-\delta_j/(\delta_j+1)}(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})\}^{1/\delta_j}$$

$$=P_{Vj}P_{Vj}^{-1/(\delta_j+1)}(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})^{1/\delta_j}$$

$$+W_LW_L^{-1/(\delta_j+1)}(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})^{1/\delta_j}$$

$$=(P_{Vj}^{\delta_j/(\delta_j+1)}+W_L^{\delta_j/(\delta_j+1)})^{(\delta_j+1)\delta_j} \tag{8}$$

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