

# The Economics of Japanese Higher-Educational Reform: Game-Theoretic Approach

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## Introduction

In Japan, the reform of higher education initiated by the Japanese government has been under way. The purpose of this reform is to introduce more competition into the Japanese higher education, encourage technological innovations, and finally construct the US-type vigorous economy. It is known that the expenditure of Japanese government to the higher education has been the lowest among the advanced countries. It is 0.9% of the Japanese National Income, while the corresponding figures of other advanced countries are as follows: the US, 1.4%, Britain, 1.8%, France, 1.2%, and Germany, 2.0% (The Ministry of Education and Science Report [2002]). While the Japanese government intends to expand the expenditure, it is also known that the Japanese universities are not efficient, in the sense that there are far less competition than in other countries. There is an agreement of opinions that reforms are required in the Japanese higher education.

In this reform, all the national universities were reconstructed in 2004 as “agencies” of British type: the National University Corporation. One of the

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methods of achieving the above goal in the new “agency” system is to rigorously evaluate the academic activity of each university, as well as expanding the financial funds to universities on a competitive basis. As for the rigorous evaluation, under the new system, the universities must submit the mid-term (time span of 6 years) plan. In the final year, the academic performance is rigorously evaluated, and the expenditure afterwards is adjusted according to the evaluation. As for the expansion of funds on a competitive basis, in 2002, the Japanese government introduced the 21st Century COE (Center of Excellence) program, which selects excellent research projects proposed by universities and provides huge amount of money on the selected excellent research institutions. In 2003, she extended her program to include the excellent educational methods.

The aim of this paper is to analyze this government policy in economic framework, extending the contribution by Bowen [1980]. Bowen [1980] constructed a simple 1-university behavior model, which maximizes prestige, not profit, under budget constraint. In this paper, his model is extended to 2-identical-university model, and Nash-type non-cooperative game is constructed in terms of *Mathematica* simulation. (This paper was originally written as *Mathematica* notebook. Since it was transformed into Word style, lots of output forms in *Mathematica* notebook are programmed not to appear on the surface for the purpose of typographic easiness. For the readers, interested in viewing those output forms, see my original *Mathematica* notebook, Fukiharu [2003], in my homepage (<http://home.hiroshima-u.ac.jp/fukito/index.htm/English>).

## I. Bowen Model and Its Extension: Simulation 1

Bowen [1980] constructed a simple 1-university behavior model, which

maximizes prestige, not profit, under budget constraint. In his model, prestige, or academic level,  $P$ , is a function of the quantity and quality of the teaching,  $T$ , and research,  $R$ , activity, undertaken by the university, and it is assumed that  $P=TR$ . The unit cost of teaching is denoted by  $c_t$ , while that of research is denoted by  $c_r$ . According to Bowen [1980], this university maximizes prestige under budget constraint,  $c_tT+c_rR=B$ . An interesting feature in Bowen model is that the budgetary fund  $B$  is assumed to be a function of  $P$ ;  $B=B(P)$ , and  $B$  is an increasing function of  $P$ .  $B(P)$  is called the donating function. He stipulates that  $B(P)=P^\alpha$ ,  $0<\alpha<1/2$ , which implies that although the university behaves as a prestige maximizer, it can raise the budgetary fund if the prestige increases, since government, firms, and students provide it with more fund. Thus, the university behaves as a prestige maximizer, taking account of this factor. It is easy to solve this maximization problem, and the optimum solution is given by  $R^*=[(c_r/c_t)^\alpha/2c_r]^{1/(1-2\alpha)}$  and  $T^*=c_rR^*/c_t$ . If  $(c_r/c_t)^\alpha/2c_r>1$ , as  $\alpha$  approaches  $1/2$ ,  $R^*$  and  $T^*$  becomes infinite.

In this paper, Bowen model is extended to 2-university behavior model. Suppose that there are 2 universities, University A and University B. Each university maximizes prestige, under budget constraint. In this model, it is assumed that the total fund for the universities is fixed by  $K$ . The government allocates the fund for each university according to the share of evaluation constructed from the donating function. Specifically, University A has prestige  $P_A$ , which is a function of the quantity and quality of the teaching,  $T_A$ , and research,  $R_A$ , activity, undertaken by the university, and it is assumed that  $P_A=T_AR_A$ . Donating function is the same as before. University A maximizes  $P_A=T_AR_A$  s.t.  $c_tT_A+c_rR_A=B_A$ , where  $B_A=KB(P_A)/(B(P_A)+B(P_B))$ , where  $P_B$  is the prestige of university B, and  $B(P)$  is the same

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function as in the original Bowen model. In the same way, University B maximizes  $P_B = T_B R_B$  s.t.  $c_l T_B + c_r R_B = B_B$ , where  $B_B = K B(P_B) / (B(P_A) + B(P_B))$ . It must be noted that in this paper  $c_l$  and  $c_r$  are the same for the both universities. Thus, 2-*identical*-university behavior model is analyzed in the game theoretic framework. University A maximizes her prestige given  $P_B$ . The optimal prestige is a function of  $P_B$ :  $P_A = \phi_A(P_B)$ , which is the reaction function of University A. University B maximizes her prestige given  $P_A$ . The optimal prestige is a function of  $P_A$ :  $P_B = \phi_B(P_A)$ , which is the reaction function of University B. Equilibrium in 2-*identical*-university behavior model is Nash non-cooperative solution; i.e.  $\{P_A^*, P_B^*\}$  which satisfies  $P_A^* = \phi_A(P_B^*)$  and  $P_B^* = \phi_B(P_A^*)$ . The increase of  $\alpha$  implies “more competition”, since if  $P_A > P_B$  holds, University A receives more fund out of  $K$  than before and University B receives less fund out of  $K$  than before.

Formerly, University A’s behavior is stipulated by

$$\begin{aligned} \max P_A = T_A R_B \quad \text{s.t.} \quad c_l T_A + c_r R_A = K B(P_A) / (B(P_A) + B(P_B)) \\ \text{where } B(P) = P^\alpha \end{aligned} \quad (1)$$

given  $P_B$ . It is not easy to solve analytically the maximizing problem in (1). Thus, we specify the parameters in (1): throughout this paper,  $c_l = c_r = 1$ . In this section it is assumed that  $K = 100$  and  $\alpha = 4/10$ . As an exercise, we compute the optimum  $P_A$  given  $P_B = 2$ . Since Lagrangean method is utilized, maximand is stipulated as  $f$  where  $r$  is the Lagrangean multiplier.

$$\begin{aligned} In[1] := & \mathbf{f} = \mathbf{R} \mathbf{A}^* \mathbf{T} \mathbf{A} - \mathbf{r}^* (\mathbf{c} \mathbf{r}^* \mathbf{R} \mathbf{A} + \mathbf{c} \mathbf{t}^* \mathbf{T} \mathbf{A} - \mathbf{K}^* (\mathbf{R} \mathbf{A}^* \mathbf{T} \mathbf{A})^\wedge(\mathbf{a}) / \\ & ((\mathbf{R} \mathbf{A}^* \mathbf{T} \mathbf{A})^\wedge(\mathbf{a}) + \mathbf{P} \mathbf{B}^\wedge(\mathbf{a}))) / \\ & \{\mathbf{a} \rightarrow 4/10, \mathbf{c} \mathbf{t} \rightarrow 1, \mathbf{c} \mathbf{r} \rightarrow 1, \mathbf{K} \rightarrow 100, \mathbf{P} \mathbf{B} \rightarrow 2\}; \end{aligned}$$

We must find the optimum  $T_A$  and  $R_A$  and  $r$ , by solving the following set of partial derivative conditions,  $d$ .

$$In[2] := \mathbf{d} = \{\mathbf{D}[\mathbf{f}, \mathbf{R} \mathbf{A}] = 0, \mathbf{D}[\mathbf{f}, \mathbf{T} \mathbf{A}] = 0, \mathbf{D}[\mathbf{f}, \mathbf{r}] = 0\};$$

It is possible to compute them approximately by means of Newton method (*Mathematica* command function is FindRoot) where the initial positions in this search are  $T_A=20$ ,  $R_A=10$  and  $r=10$ .

```
In[3]:= sol2=FindRoot[d, {RA, 20}, {TA, 10}, {r, 10}]
```

```
Out[3]= {RA -> 47.1483, TA -> 47.1483, r -> 49.4024}
```

It is checked in what follows that sol2 is actually maximum, not minimum. In order to do so, the following constraint equation is solved for  $R_A$  as a function of  $T_A$ .

```
In[4]:= k1=D[f, r]; sol1=Solve[k1==0, RA];
```

It has 7 solutions as shown in what follows.

```
In[5]:= Length[sol1]
```

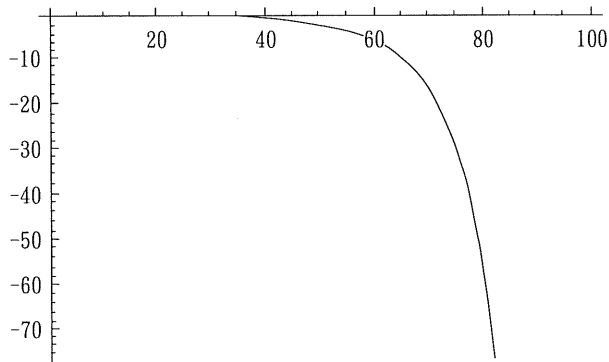
```
Out[5]= 7
```

In order to show the graph of  $P_A = T_A R_A$  under constraint, define f1 as follows.

```
In[6]:= f1=(RA*TA)/.sol1;
```

Among these 7 solutions, the 1st solution gives rise to negative values of  $f1 = R_A T_A$  as shown by the following graph.

```
In[7]:= Plot[f1[[1]], {TA, 0, 100}];
```



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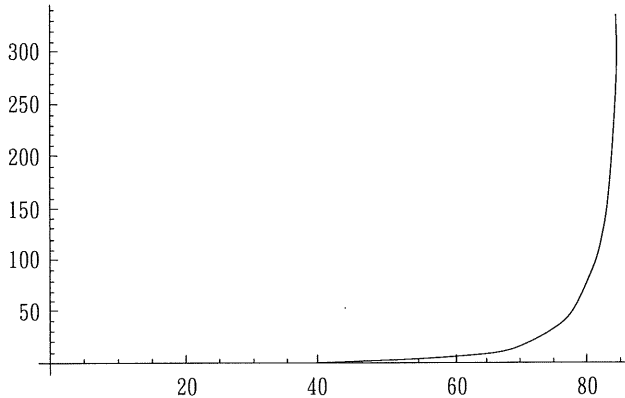
Indeed, when  $TA = 50$ ,  $RA$  is negative, as shown in what follows.

```
In[8]:= N[sol1[[1]]/.TA→50]
```

```
Out[8]= {RA → -0.039841}
```

Among these 7 solutions, the 2nd solution gives rise to positive value of  $f_1$ . This solution, however, cannot exceed 400, as shown in what follows.

```
In[9]:= Plot[f1[[2]], {TA, 0, 100}];
```



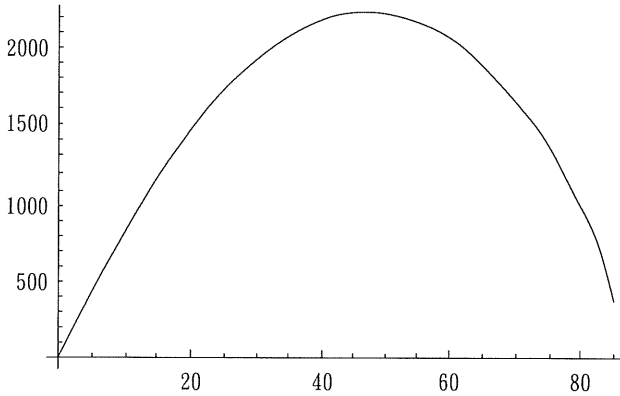
```
In[10]:= {N[f1[[2]]/.TA→84.62], N[f1[[2]]/.TA→84.621]}
```

```
Out[10]= {336.589, 340.644 - 5.01398 I}
```

where I is “Imaginary I”.

Among these 7 solutions, the 3rd solution gives rise to positive values of  $f_1 = RA TA$ . This solution seems to have maximum at  $TA = 47.1483$ , as shown in what follows.

```
In[11]:= Plot[f1[[3]], {TA, 0, 100}];
```



In this paper, a simulation approach is adopted. In other words, Newton method is utilized, in which solutions to equations may not be found depending on the initial positions. As shown in what follows, however, the Newton method allows us to find the solution  $P_A$  which satisfies the first order conditions when  $P_B=2000$ , even if the same initial positions;  $T_A=20$ ,  $R_A=10$  and  $r=10$ , are selected.

```
In[12]:= f=RA*TA-r*(cr*RA+ct*TA-K*(RA*TA)^(a)/
((RA*TA)^(a)+PB^(a))/.{a→4/10, ct→1, cr→1, K→100,
PB→2000}; d={D[f, RA]=0, D[f, TA]=0, D[f, r]=0};
FindRoot[d, {RA, 20}, {TA, 10}, {r, 10}]
```

```
Out[12]= {RA → 14.3652, TA → 14.3652, r → 33.4196}
```

Thus, the Newton method may allow us to find the optimum  $P_A$  for  $P_B=1, 2, \dots, 2000$ , using the same initial positions;  $T_A=20$ ,  $R_A=10$  and  $r=10$ . If this approach is successfully adopted, the obtained pairs  $\{P_B, P_A\}$  represent the reaction function of the University A. Indeed, these optimum  $P_A$  can be found by using the following function, checkA, for  $P_B=1, 2, \dots, 2000$ . To be precise, checkA computes the optimum  $P_A$  with the same initial positions;  $T_A=20$ ,  $R_A=10$  and  $r=10$ , when  $\alpha, c_i, c_r, K$ , and  $P_B$  are given arbitrarily.

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```
In[13]:= checkA[a0_, ct0_, cr0_,k0_, pb0_] :=
Module[{f, sol2, d}, f=RA*TA-r*(cr*RA+ct*TA-
K*(RA*TA)^(a)/((RA*TA)^(a)+PB^(a)))/.
{a→a0, ct→ct0, cr→cr0, K→k0, PB→pb0};
d={D[f, RA]==0, D[f, TA]==0, D[f, r]==0};
sol2=FindRoot[d, {RA, 20}, {TA, 10}, {r, 10}];
(RA/.sol2)*(TA/.sol2)]
```

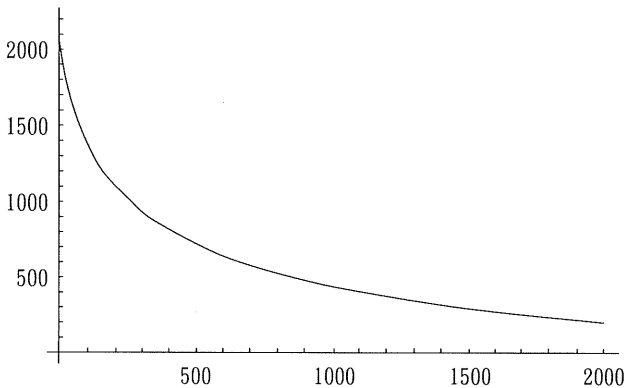
2000 pairs of  $\{P_B, P_A\}$ , A1, can be found using checkA, in approximately 22 seconds.

```
In[14]:= Timing[A1=Table[{pb0, checkA[4/10, 1, 1, 100, pb0]},
{pb0, 2, 2000}];]
```

```
Out[14]= {21.601 Second, Null}
```

A1 is plotted in the following fig11.

```
In[15]:= fig11=ListPlot[A1, PlotJoined->True];
```



In this paper, for the ease of computation, symmetry assumption is adopted; *i. e.* 2 universities have the same unit costs in conducting research and teaching. Thus, for the same  $c_t$  and  $c_r$ , university B's behavior is stipulated by

$$\max P_B = T_B R_B \quad \text{s.t.} \quad c_t T_B + c_r R_B = K B(P_B) / (B(P_A) + B(P_B))$$



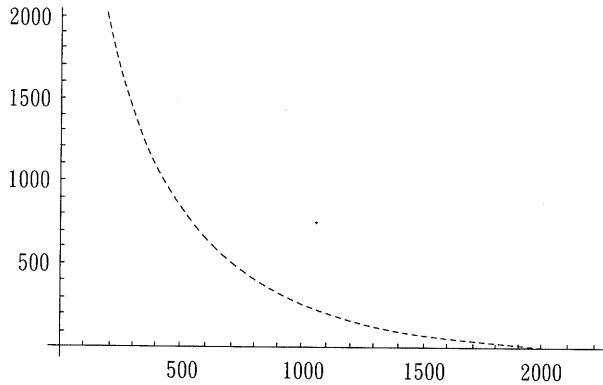
$$\text{where } B(P) = P^\alpha \quad (2)$$

given  $P_A$ . Note that (1) and (2) are symmetrical. We specify the same parameters in (2) as in (1). Furthermore, throughout this section,  $c_t = c_r = 1$ ,  $K = 100$  and  $\alpha = 4/10$  as in (1). Then, if we change the position of  $P_B$  and  $P_A$  in A1, the set of 2000 pairs, B1, is the reaction function for University B, given  $P_A$ .

```
In[16]:= B1=Table[{A1[[i, 2]], A1[[i, 1]]}, {i, 1, Length[A1]}];
```

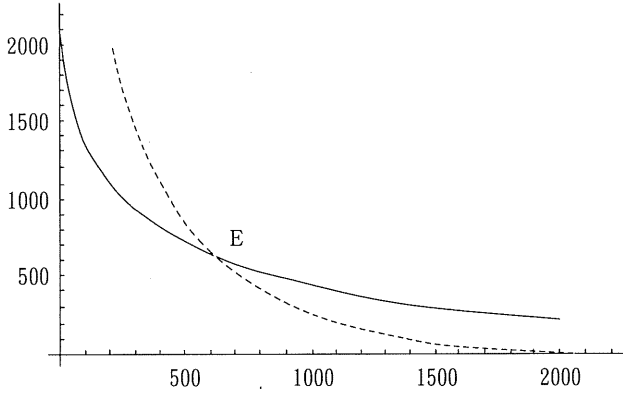
B1 is plotted in the following fig12.

```
In[17]:= fig12=ListPlot[B1, PlotJoined→True,
PlotStyle→Dashing[{0.01}]];
```



Putting them together, the intersection of the 2 reaction functions, E, is the Nash non-cooperative solution to the prestige game. This solution is stable as shown in the following figure.

```
In[18]:= Show[fig11, fig12];
```



The Nash-noncooperative-game solution E can be found by solving  $P_B = TARA$  and the first order conditions, as done in what follows.

$$\begin{aligned} \text{In}[19] := & \text{f1} = \text{RA} * \text{TA} - \text{r} * (\text{cr} * \text{RA} + \text{ct} * \text{TA} - \text{K} * (\text{RA} * \text{TA})^{\text{a}}) / \\ & ((\text{RA} * \text{TA})^{\text{a}} + \text{PB}^{\text{a}}) /. \{ \text{a} \rightarrow 4/10, \text{ct} \rightarrow 1, \text{cr} \rightarrow 1, \text{K} \rightarrow 100 \}; \\ & \text{FindRoot}[\{ \text{TA} * \text{RA} = \text{PB}, \text{D}[\text{f1}, \text{RA}] = 0, \text{D}[\text{f1}, \text{TA}] = 0, \\ & \text{D}[\text{f1}, \text{r}] = 0 \}, \{ \text{RA}, 20 \}, \{ \text{TA}, 10 \}, \{ \text{r}, 10 \}, \{ \text{PB}, 500 \}] \end{aligned}$$

$$\text{Out}[19] = \{ \text{RA} \rightarrow 25., \text{TA} \rightarrow 25., \text{r} \rightarrow 41.6667, \text{PB} \rightarrow 625. \}$$

It was shown that at the Nash solution E,  $P_B = P_A = 625$ .

## II. Comparison with Less Competitive Policy:

### Simulation 2

In this section, simulation in the former section is modified to the one in which  $\alpha$  is smaller than in the former section: less competitive case. University A's behavior is stipulated by (1) given  $P_B$ , where we specify the parameters in (1) by the following:  $c_t = c_r = 1, K = 100$  and  $\alpha = 1/10$ . As an exercise, we compute the optimum  $P_A$  given  $P_B = 2$ . Since Lagrangean method is utilized, maximand is stipulated as  $f_2$ . We must find the optimum  $T_A$  and  $R_A$  and  $r$ , by solving the following set of partial derivative conditions, d.

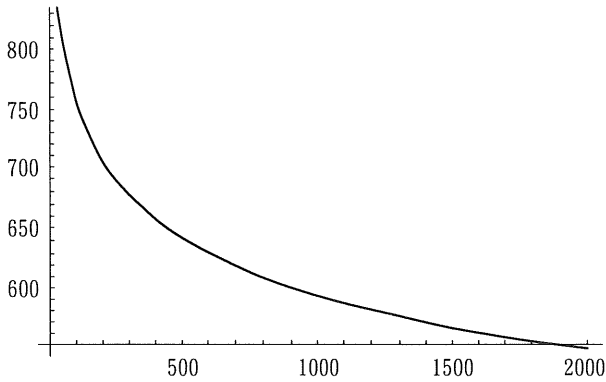
Then, in exactly the same way as in section I, it is possible to compute the optimal values approximately by means of Newton method where the initial positions in this search are  $T_A=20$ ,  $R_A=10$  and  $r=10$ .

```
In[20]:= f2=RA*TA-r*(cr*RA+ct*TA-K*(RA*TA)^(a)/
(((RA*TA)^(a)+PB^(a)))/.{a→1/10, ct→1,
cr→1, K→100,PB→2};
d={D[f2, RA]=0, D[f2, TA]=0, D[f2, r]=0};
sol2=FindRoot[d, {RA, 20}, {TA, 10}, {r, 10}]
```

```
Out[20]= {RA → 32.5964, TA → 32.5964, r → 35.0354}
```

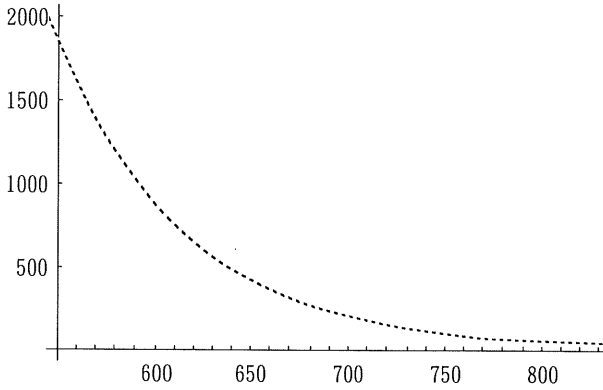
As in section I, the Newton method allows us to find the optimum  $P_A$  for  $P_B=1, 2, \dots, 2000$ , using the same initial positions;  $T_A=20$ ,  $R_A=10$  and  $r=10$ . This approach is successfully adopted, and the obtained pairs  $\{P_B, P_A\}$  represent the reaction function of the University A. Indeed, these optimum  $P_A$  can be found by using the function constructed in section I, checkA, for  $P_B=1, 2, \dots, 2000$ . 2000 pairs of  $\{P_B, P_A\}$ , A2, can be found using checkA, in approximately 23 seconds. A2 is plotted in the following fig21.

```
In[21]:= A2=Table[{pb0, checkA[1/10, 1, 1, 100, pb0]}, {pb0, 1, 2000}];
fig21=ListPlot[A2, PlotJoined→True, PlotStyle→
Thickness[0.01]];
```



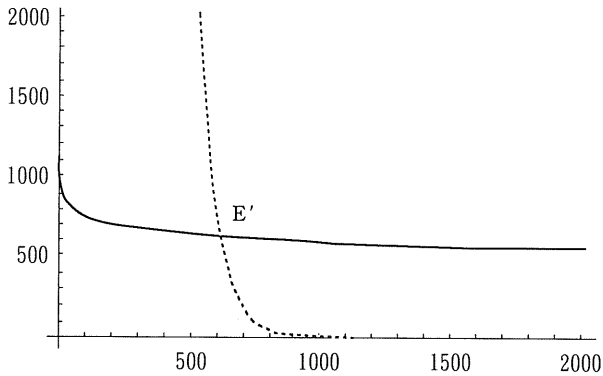
As in section I, for the ease of computation, symmetry assumption is adopted; University B's behavior is stipulated by (2) given  $P_A$ . As mentioned in section I, (1) and (2) are symmetrical. We specify the same parameters in (2) as in (1): throughout this section,  $c_t = c_r = 1$ ,  $K = 100$  and  $\alpha = 1/10$  are assumed as in (1). Then, if we change the position of  $P_B$  and  $P_A$  in A2, the set of 2000 pairs, B2, is the reaction function for University B, given  $P_A$ . B2 is plotted in the following fig22.

```
In[22]:= B2=Table[{A2[[i, 2]], A2[[i, 1]]}, {i, 1, Length[A2]}];
fig22=ListPlot[B2, PlotJoined→True,
PlotStyle→{Dashing[{0. 015}], Thickness[0. 01]}];
```



Putting them together, the intersection of the two reaction functions,  $E'$ , is the Nash non-cooperative solution to the prestige game. This solution,  $E'$ , is stable as shown in the following figure.

```
In[23]:= figt2=Show[Graphics[Text["E'", {700, 700}]],  
DisplayFunction->Identity];  
Show[{fig21, fig22, figt2}, DisplayFunction->  
$DisplayFunction];
```



Note that the Nash solutions,  $E$ , and  $E'$  are stable and  $E$  could be computed in section I. Likewise,  $E'$  can be found by solving  $P_B = T_{A}R_A$  and the

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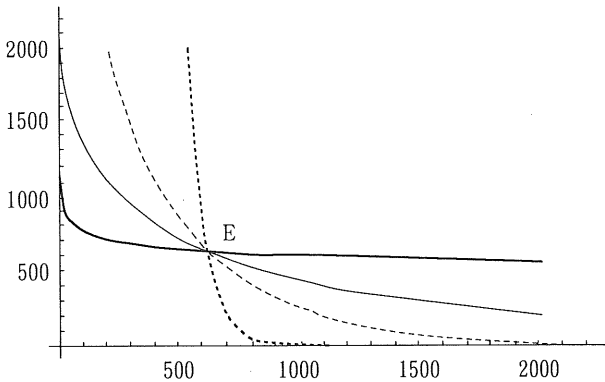
set of first order conditions, as done in what follows.

```
In[24]:= f1=RA*TA-r*(cr*RA+ct*TA-K*(RA*TA)^(a)/
((RA*TA)^(a)+PB^(a)))/. {a->1/10, ct->1, cr->1, K->100};
FindRoot[{TA*RA=PB, D[f1, RA]=0,
D[f1, TA]=0, D[f1, r]=0}, {RA, 20}, {TA, 10},
{r, 10}, {PB, 500};
```

```
Out[24]= {RA -> 25., TA -> 25., r -> 27.7778, PB -> 625.}
```

It was shown that at the Nash solution  $P_B = P_A = 625$ ; the same Nash solution as in section I, or  $E = E'$ . This situation is depicted in the following diagram, by putting together four diagrams, constructed so far.

```
In[25]:= Show[{fig11, fig12, fig21, fig22, figt1},
DisplayFunction->$DisplayFunction];
```

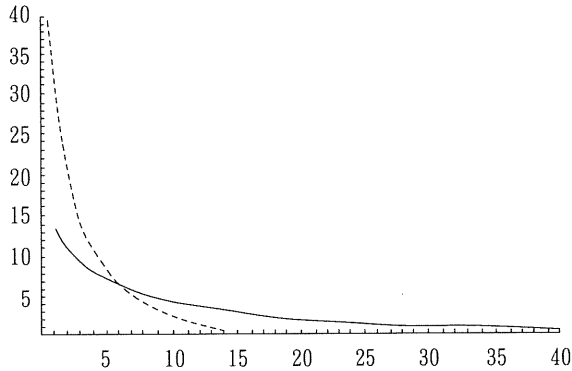


The conclusion so far,  $E = E'$ , implies that even if the government modifies its educational policy so that more competition is introduced into the higher education, the modified policy is ineffective so long as the two universities are identical in conducting research and teaching. This conclusion appears to be natural.

In order to confirm the robustness of the simulation, let us specify the

parameters in (1) as  $c_l = c_r = 1$ ,  $K = 10$  and . As above, it is easy to construct the reaction functions of University A and University B, and depict them in a diagram by the following commands.

```
In[26]:= A3=Table[{pb0, checkA[4/10, 1, 1, 10, pb0]}, {pb0, 1, 2000}];
fig31=ListPlot[A3, PlotJoined→True,
DisplayFunction→Identity];
B3=Table[{A3[[i, 2]], A3[[i, 1]]}, {i, 1, Length[A3]}];
fig32=ListPlot[B3, PlotJoined→True,
PlotStyle→Dashing[{0.01}], DisplayFunction→Identity];
Show[{fig31, fig32}, DisplayFunction→$DisplayFunction,
PlotRange→{{0, 40}, {0, 40}}];
```



Note that the Nash solution is stable as above. This solution can be found by solving  $P_B = T_A R_A$  and the set of first order conditions, as done in what follows.

```
In[27]:= f10=RA*TA-r*(cr*RA+ct*TA-K*(RA*TA)^(a)/
((RA*TA)^(a)+PB^(a))/. {a→4/10, ct→1, cr→1, K→10};
FindRoot[{TA*RA==PB, D[f10, RA]=0, D[f10, TA]=0,
D[f10, r]=0}, {RA, 2}, {TA, 3}, {r, 5}, {PB, 6}]
```

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*Out[27]=* {RA → 2.5, TA → 2.5, r → 4.16667, PB → 6.25}

What would happen if  $\alpha$  is changed from 4/10 to 1/10 as above? We have the same Nash-non-cooperative-game solution as in  $\alpha=4/10$ , as done in what follows.

*In[28]:=* f11=RA\*TA-r\*(cr\*RA+ct\*TA-K\*(RA\*TA)^(a)/  
 ((RA\*TA)^(a)+PB^(a))/.{a→1/10, ct→1, cr→1, K→10};  
**FindRoot**[{TA\*RA=PB, D[f11, RA]=0, D[f11, TA]=0,  
 D[f11, r]=0}, {RA, 2}, {TA, 3}, {r, 5}, {PB, 6}]

*Out[28]=* {RA → 2.5, TA → 2.5, r → 2.77778, PB → 6.25}

Thus, we have the same conclusion when  $K=10$ ; even if the government modifies its educational policy so that more competition is introduced into the higher education, the modified policy is ineffective so long as the two universities are identical in conducting research and teaching.

### III. Non-Existence Case in the Too-Much-Competitive Policy: Simulation 3

In this section, the existence and instability problem is examined; if the government selects large  $\alpha$ , is the existence and stability of Nash-non-cooperative solution guaranteed? In order to examine this problem in terms of *Mathematica* simulation, suppose that  $\alpha=1$ , with the same stipulation on  $c_t$ ,  $c_r$ , and  $K$ ;  $c_t=c_r=1$ ,  $K=100$ . In this simulation, we can analytically solve the maximizing problem as in what follows. To see this suppose that  $P_B=2$ , in computing optimal  $P_A$ .

*In[29]:=* f=RA\*TA-r\*(cr\*RA+ct\*TA-K\*(RA\*TA)^(a)/  
 ((RA\*TA)^(a)+PB^(a))/.{a→1, ct→1, cr→1, K→100, PB→2};  
**d**={D[f, RA]=0, D[f, TA]=0, D[f, r]=0};  
**sol2**=Solve[d, {RA, TA, r}]; N[sol2]



```
Out[29]= {{r → 0., RA → 0., TA → 0.},
           {r → 50.0401, RA → 49.96, TA → 49.96},
           {r → -0.0400963, RA → 0.0400321, TA → 0.0400321}}
```

The second solution is the required one. Thus, *Mathematica* command function, `Solve`, may allow us to find the optimum  $P_A$  for  $P_B=1, 2, \dots, 2000$ . If this approach is successfully adopted, the obtained pairs  $\{P_B, P_A\}$  represent the reaction function of the University A. Indeed, these optimum  $P_A$  can be found by using the following function, `checkAA`, for  $P_B=1, 2, \dots, 2000$ . To be precise, `checkAA` computes the optimum  $P_A$  when  $\alpha, c_t, c_r, K$ , and  $P_B$  are given arbitrarily.

```
In[30]:= checkAA[a0_, ct0_, cr0_, k0_, pb0_] :=
Module[{f, sol2, d, sol3},
  f=RA*TA-r*(cr*RA+ct*TA-K*(RA*TA)^(a)/
  ((RA*TA)^(a)+PB^(a)))/
  {a→a0, ct→ct0, cr→cr0, K→k0, PB→pb0};
  d={D[f, RA]=0, D[f, TA]=0, D[f, r]=0};
  sol2=Solve[d, {RA, TA, r}]; sol3=Select[sol2, (r/.#)>0&];
  ((RA/.sol3)*(TA/.sol3))[[1]]]
```

If  $P_B < 625$ , it is possible to find the optimum  $P_A$  as shown in what follows, and it seems that the optimum  $P_A$  converges to 625 when  $P_B$  approaches 625.

```
In[31]:= {checkAA[1, 1, 1, 100, 624], checkAA[1, 1, 1, 100, 624.9],
          checkAA[1, 1, 1, 100, 624.998],
          checkAA[1, 1, 1, 100, 624.9999999]}

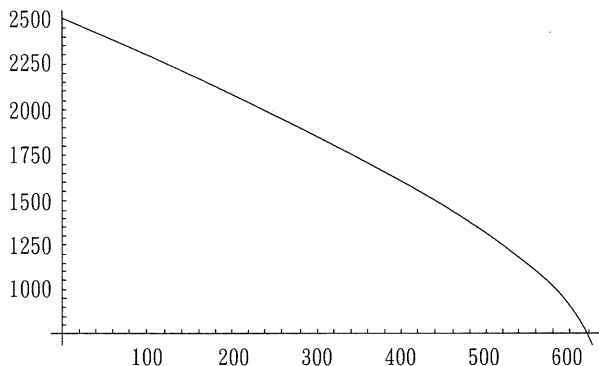
Out[31]= {676, 640.911, 627.238, 625.016}
```

When  $P_B < 624$ , optimum  $P_A$  can be found by using the function, `checkAA`, and those pairs of  $\{P_B, P_A\}$ , A4, is constructed in approximately 22 seconds.

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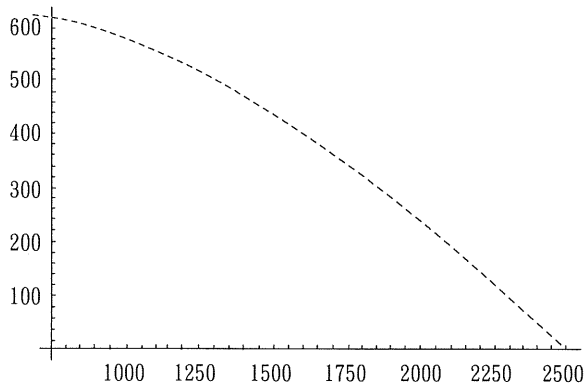
A4 is plotted in the following fig41.

```
In[32]:= A4=Table[{pb0, checkAA[1, 1, 1, 100, pb0]}, {pb0, 1, 624}];  
fig41=ListPlot[A4, PlotJoined→True];
```



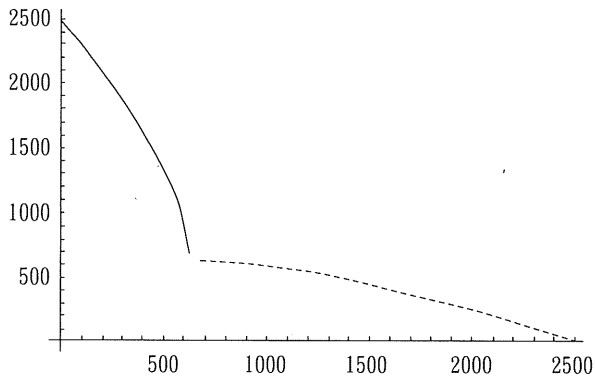
As in the previous sections, for the ease of computation, symmetry assumption is adopted; University B's behavior is stipulated by (2) given  $P_A$ . As mentioned in the previous sections, (1) and (2) are symmetrical. We specify the same parameters in (2) as in (1): throughout this section,  $c_t = c_r = 1$ ,  $K=100$  and  $\alpha=1$  as in (1). Then, if we change the position of  $P_B$  and  $P_A$  in A4, the set of 624 pairs, B4, is the reaction function for University B, given  $P_A$ . B4 is plotted in the following fig42.

```
In[33]:= B4=Table[{A4[[i, 2]], A4[[i, 1]]}, {i, 1, Length[A4]}];  
fig42=ListPlot[B4, PlotJoined→True,  
PlotStyle→Dashing[{0.01}]];
```



Putting them together, if they intersect with each other, it is the Nash non-cooperative solution to the prestige game.

`In[34]:= Show[fig41, fig42];`



It is shown in what follows that there exists no Nash non-cooperative solution to the prestige game when  $\alpha=1$ . As done in the previous sections, the Nash solution could be found by solving  $P_B = T_A R_A$  and the set of first order conditions utilizing Newton method, so long as there exists one. However, the Newton method cannot have the convergence. In what follows, somewhat different approach is adopted to show that there exists no Nash

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non-cooperative solution to the prestige game when  $\alpha=1$ . When  $P_B = T_A R_A$  and  $P_B$  is not 0,  $D[f, r]=0$  becomes the following equation.

$$\begin{aligned} \text{In}[35] := & \mathbf{f} = \mathbf{R} \mathbf{A} * \mathbf{T} \mathbf{A} - \mathbf{r} * (\mathbf{c} \mathbf{r} * \mathbf{R} \mathbf{A} + \mathbf{c} \mathbf{t} * \mathbf{T} \mathbf{A} - \mathbf{K} * (\mathbf{R} \mathbf{A} * \mathbf{T} \mathbf{A})^{\wedge}(\mathbf{a}) / \\ & ((\mathbf{R} \mathbf{A} * \mathbf{T} \mathbf{A})^{\wedge}(\mathbf{a}) + \mathbf{P} \mathbf{B}^{\wedge}(\mathbf{a}))) / .\{\mathbf{a} \rightarrow 1, \mathbf{c} \mathbf{t} \rightarrow 1, \mathbf{c} \mathbf{r} \rightarrow 1, \mathbf{K} \rightarrow 100\}; \\ \mathbf{D}[\mathbf{f}, \mathbf{r}] = & 0 / .\mathbf{P} \mathbf{B} \rightarrow \mathbf{T} \mathbf{A} * \mathbf{R} \mathbf{A} \end{aligned}$$

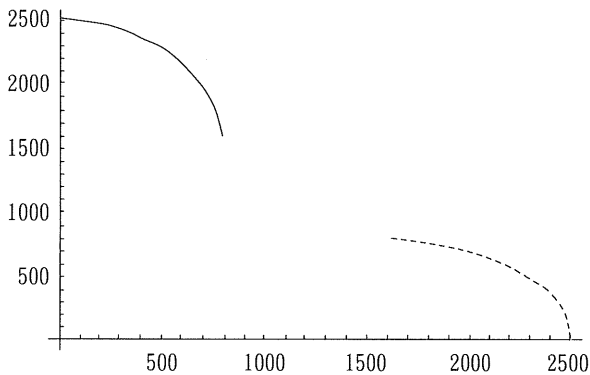
$$\text{Out}[35] = 50 - \mathbf{R} \mathbf{A} - \mathbf{T} \mathbf{A} == 0$$

Note that it is assumed that  $T_A \neq 0$  and  $R_A \neq 0$ . Substituting this condition and  $P_B = T_A R_A$  into  $D[f, T_A]=0$ , and solving this equation with respect to  $r$ , we have the following solution, solA. Substituting solA and other conditions for  $D[f, R_A]=0$ , we can solve this equation with respect to  $T_A$ .

$$\begin{aligned} \text{In}[36] := & \text{solA} = \text{Solve}[\mathbf{D}[\mathbf{f}, \mathbf{T} \mathbf{A}] = 0 / .\{\mathbf{P} \mathbf{B} \rightarrow \mathbf{T} \mathbf{A} * \mathbf{R} \mathbf{A}, \mathbf{R} \mathbf{A} \rightarrow 50 - \mathbf{T} \mathbf{A}\}, \\ & \mathbf{r}][[1]]; \\ & \text{Solve}[(\mathbf{D}[\mathbf{f}, \mathbf{R} \mathbf{A}] / .\mathbf{P} \mathbf{B} \rightarrow \mathbf{R} \mathbf{A} * \mathbf{T} \mathbf{A} / .\text{solA} / .\mathbf{R} \mathbf{A} \rightarrow 50 - \mathbf{T} \mathbf{A}) == 0, \mathbf{T} \mathbf{A}] \end{aligned}$$

$$\text{Out}[36] = \{\{\mathbf{T} \mathbf{A} \rightarrow 0\}\}$$

This is the desired contradiction since  $T_A \neq 0$  and  $R_A \neq 0$  are assumed. Therefore, there exists no Nash non-cooperative solution to the prestige game when  $\alpha=1$ . When  $\alpha=2$ , we have the following reaction functions.



### III. Conclusions

The aim of this paper is to analyze the government's educational policy in economic framework, extending the contribution by Bowen [1980]. Bowen [1980] constructed a simple 1-university behavior model, which maximizes prestige, not profit, under budget constraint. Here, the prestige may be regarded as the academic level of the university. In this paper, his model is extended to 2-identical-university model, and Nash-type non-cooperative game is constructed, where the government provides budgetary fund competitively. Casual guess may be possible; the Nash non-cooperative solution is stable and two (identical) universities prevail when the competition is not severe, and it will be unstable and one of the universities may disappear when the competition is quite severe. In examining this casual guess, it is not easy to compute reaction functions analytically, so that a simulation approach is adopted, by specifying unit costs of conducting research and teaching numerically. It is shown that the Nash non-cooperative solution is stable and two universities prevail when the competition is not severe, and there is no Nash non-cooperative solution when the competition is severe. The interesting result in this simulation is that when the stable Nash-type game solutions exist, the resulting stable prestige (academic level of the university) is independent of the government funding policy. Thus, the conclusion of this paper is that the mere introduction of competition by the government cannot enhance the academic levels of universities when the universities are *identical*. The analysis when the universities are different in the unit costs of conducting research and teaching will be done in another paper (see Fukiharu [2004]).

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