Currency Denomination and the Optimal Portfolio

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Abstract

Recent Japanese firms use foreign currency for domestic transactions. Increased use of foreign currency for it may induce a change of the currency denomination in which profit itself is measured. We studied the effect of a change of the currency denomination on an agent's portfolio behavior by using the expected utility maximization approach. Despite expectations of exchange rate fluctuation, foreign asset holdings can be a larger proportion of an agent's portfolio if returns are measured in domestic currency rather than in foreign currency. However, when the degree of risk aversion is high enough, such a situation will never occur.

1. Introduction

Choice of currency denomination has been analyzed in the context of the optimal invoicing strategies of a monopolistic exporting firm. The problem is the choice of currency in which to set price from the viewpoint of an exporting firm that maximizes the expected utility of profit in its domestic currency under exchange rate uncertainty. A common finding in the invoicing literature is that the importer's currency is preferred when a profit function is globally concave with respect to exchange rate. This result is independent of the degree of risk aversion with respect to profit (see Giovannini (1988), Donnenfeld and Zilcha (1991), Friberg (1998), Bacchetta and Wincoop (2002)).

Recently some Japanese firms have started to use foreign currency not only for international transactions but also for domestic transactions.[1] An increase in the use of foreign currency for domestic transactions may induce a change of the currency denomination in which profit itself is measured. Such a change may have effects on a firm's optimal portfolio behavior. This problem has already been investigated by Okishio (1989), who uses a simple model to analyze whether or not a firm's choice of investment and borrowing activity depends on the currency denomination. He suggested the paradoxical case that even a risk-averse firm can choose a risky foreign project when measuring profit in domestic currency. This result relates to a property of expectations that causes the so-called Siegel paradox.[2]

In this paper, we use the expected utility maximization approach to examine the sufficient conditions under which the change of currency denomination from domestic to foreign currency increases foreign currency asset holdings. Our approach differs from the invoicing literature in dealing with both expected utility in domestic currency and expected utility in foreign currency, and it emphasizes the role of risk aversion.

In section 2, we present our model, and in section 3, we discuss the sufficient condition that foreign currency asset holdings increase after the change of currency denomination from domestic currency to the foreign currency. In section 4, we show the role of risk aversion by using a spe-

(1) In Japan, the use of foreign currency for payments between domestic firms was liberalized when the foreign exchange and foreign trade law was amended in 1998.

(2) See Siegel (1972), Obstfeld and Rogoff (1996) 586-588.

cific utility function.

2. Portfolio problem

Consider a risk-averse agent that invests its resources, W, in domestic and foreign currency assets. The initial exchange rate, defined as the price of foreign currency in terms of domestic currency, is normalized to unity. Therefore, initially, the value of assets measured in domestic currency is the same as that measured in foreign currency. Let us assume that an agent maximizes expected utility from nominal returns.(3) If an agent denominates nominal returns in domestic currency, it will solve the following problem.

$$\max E[U\{(1+r)(W-x)+(1+r^*)ex\}] \qquad (1)$$
$$0 \leq x \leq W$$

where U is the twice continuously differentiable utility function with $U'>0$, $U''<0$; x denotes the value of foreign currency assets the agent holds; and r and r^* denote the interest rates on domestic and foreign currency assets respectively. We assume that both foreign and domestic interest rates are constant and that the exchange rate at the end of the period, denoted e, is a continuous random variable with probability density function $f(e)$.

(3) When an agent maximizes real returns in terms of a well-behaved price index, currency denomination does not matter. Let us define the domestic and foreign price indexes as

$$P=p^{\theta}(ep^*)^{1-\theta}, \quad P^*=\left(\frac{p}{e}\right)^{\theta}(p^*)^{1-\theta}$$

where p and p^* are the prices of domestic goods and foreign goods in terms of each country's own currency. When an agent evaluates returns in terms of these price indexes, real returns are independent of currency denomination. However, we assume that an agent is interested in nominal returns.

If an agent denominates nominal returns in foreign currency, it will solve the following problem.(4)

$$\max \mathrm{E}\left[U\left\{\frac{(1+r)(W-x)}{e}+(1+r^{*})x\right\}\right] \quad (2)$$

$$0 \leq x \leq W$$

When the currency denomination is the domestic currency, foreign currency assets are risky. A fluctuation in the exchange rate has an effect only on returns from foreign currency assets. In this case, depreciation in the foreign currency decreases returns. On the other hand, when returns are measured in terms of the foreign currency, domestic currency assets are risky assets and an appreciation in foreign currency decreases returns.

Intuition would suggest that the optimal amount of foreign currency assets would be greater if returns were measured in foreign currency than if measured in domestic currency. To test this intuition, let us compare x_1^* and x_2^*, which are solutions for (1) and (2) respectively.

The Kuhn-Tucker conditions are as follows.(5)

For the problem (1),

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(4) When an agent expects a change in exchange rate, it will not think that the expected utility of one unit of the domestic currency and the expected utility of one unit of the foreign currency are the same. The following definition of expected utility of foreign currency returns is a simple way to compare these two expected utilities.

$$E\left[U\left\{\frac{(1+r)(W-x)}{e}+(1+r^{*})x\right\}\right]E[e]$$

That is, when an agent expects that the foreign currency will be worth $E[e]$ times what is now, it multiplies the expected utility of the foreign currency returns by $E[e]$. Since $E[e]$ is certain, the agent which denominates nominal returns in the foreign currency will solve problem (2).

(5) See Appendix (A).

$$If\ x_1^* = 0, \qquad then\ E(e) \leq \frac{1+r}{1+r^*} \tag{3}$$

$If\ 0 < x_1^* < W,$

$$then\ E[\{-(1+r)+(1+r^*)e\}U'\{(1+r)(W-x_1^*)+(1+r^*)ex_1^*\}] = 0 \tag{4}$$

(4) implies

$$\frac{1+r}{1+r^*} < E[e].$$

$$If\ x_1^* = W, \qquad then\ \frac{1+r}{1+r^*} + \alpha \leq E[e] \tag{5}$$

$$where\ \alpha = \frac{-\mathrm{cov}[e, U'\{(1+r^*)eW\}]}{E[U'\{(1+r^*)\}eW]} > 0$$

For the problem (2),

$$If\ x_2^* = 0, \qquad then\ \frac{1+r^*}{1+r} + \beta \leq E\left[\frac{1}{e}\right] \tag{6}$$

$$where\ \beta = \frac{-\mathrm{cov}\left[\frac{1}{e}, U'\left\{\frac{(1+r)W}{e}\right\}\right]}{E\left[U'\left\{\frac{(1+r)W}{e}\right\}\right]} > 0$$

$If\ 0 < x_2^* < W,$

$$then\ E\left[\left\{\frac{-(1+r)}{e} + (1+r^*)\right\}U'\left\{\frac{(1+r)(W-x_2^*)}{e} + (1+r^*)x_2^*\right\}\right] = 0 \tag{7}$$

(7) implies

$$\frac{1+r^*}{1+r} < E\left(\frac{1}{e}\right)$$

$$If\ x_2^* = W, \qquad then\ E\left(\frac{1}{e}\right) \leq \frac{1+r^*}{1+r} \tag{8}$$

Since the objective functions are concave with respect to foreign currency assets holdings, the Kuhn-Tucker conditions are necessary and sufficient conditions.

When $x_1^* = 0$ or $x_2^* = W$, $x_1^* \leq x_2^*$ always holds. In these cases, a depreciation of the one currency always implies an appreciation of the other currency without reference to currency denomination. That is, $E[e] \leq (1+r)/(1+r^*)$ means $(1+r^*)/(1+r) \leq E(1/e)$, and $E(1/e) \leq (1+r^*)/(1+r)$ means $(1+r)/(1+r^*) \leq E(e)$. This follows from Jensen's inequality.(6)

However, when $x_1^*=W$ or $x_2^*=0$, $x_1^* \leq x_2^*$ does not always hold. When $x_1^*=W$, the following holds.

$$\frac{1+r}{1+r^*} < \frac{1+r}{1+r^*} + \alpha \leq E[e]$$

Therefore,

$$\frac{1}{E[e]} < \frac{1+r^*}{1+r}.$$

Since $1/E[e] < E[1/e]$, it is not clear that $E[1/e]$ is less than $(1+r^*)/(1+r)$. If $(1+r^*)/(1+r) < E[1/e]$, then $x_2^* \neq W$, that is, $x_1^* > x_2^*$. This is the case where both currencies are expected to appreciate. This phenomenon can occur when the variance is large enough.(7)

When $x_2^*=W$, the same thing can occur. When both inequalities $(1+r)/(1+r^*) < E[e]$ and $(1+r^*)/(1+r) < E[1/e]$ hold, x_1^* is lager than x_2^* at the corner solutions.

As for inner solutions, x_1^* or x_2^* may be larger. It will depend not only on the density function but also on the utility function.

3. A comparison of two inner solutions

In this section, we assume inner solutions for both problems (1) and (2). To see whether x_1^* or x_2^* is larger under this condition, we shall substitute x_1^* into the left side of (7). If it is positive, x_2^* is larger than x_1^*, since

(6) Jensen's inequality is $1/E[e] < E[1/e]$.

(7) See Appendix (B).

the second derivative of expected utility with respect to foreign currency assets is negative.

From (4), x_1^* must satisfy equation (9).

$$E\left[\frac{MU_2(e)MU_1(e)}{MU_2(e)}\right]=0 \tag{9}$$

where

$$MU_1(e)=\{-(1+r)+(1+r^*)e\}U'\{(1+r)(W-x_1^*)+(1+r^*)ex_1^*\} \tag{10}$$

$$MU_2(e)=\left\{\frac{-(1+r)}{e}+(1+r^*)\right\}U'\left\{\frac{(1+r)(W-x_1^*)}{e}+(1+r^*)x_1^*\right\} \tag{11}$$

Here, $E[MU_2(e)]$ is the left hand side of (7) at $x=x_1^*$.

An alternative way to express (9) is

$$E[MU_2(e)]E\left[\frac{MU_1(e)}{MU_2(e)}\right]+\text{cov}\left[MU_2(e), \frac{MU_1(e)}{MU_2(e)}\right]=0 \tag{12}$$

Since $0<E\left[\frac{MU_1(e)}{MU_2(e)}\right]$, we obtain

$$\text{cov}\left[MU_2(e), \frac{MU_1(e)}{MU_2(e)}\right]\gtreqless 0 \leftrightarrow E[MU_2(e)]\gtreqless 0 \leftrightarrow x_1^*\gtreqless x_2^* \tag{13}$$

The sign of the covariance is ambiguous because an appreciation of the foreign currency may raise both MU_1 and MU_2.

Let us define $H(e)$ as $MU_1(e)/MU_2(e)$. $H(e)$ is rewritten as follows.

$$H(e)=e\frac{U'\{(1+r)(W-x_1^*)+(1+r^*)ex_1^*\}}{U'\left\{\frac{(1+r)(W-x_1^*)}{e}+(1+r^*)x_1^*\right\}}$$

Intuitively, when the curvature of the utility function is sharp, the covariance tends to be negative. Thus, we may infer that high risk aversion corresponds with negative covariance. We shall examine this point by specifying a utility function later.

$H'(e)<0$ is a sufficient condition for x_2^* to be larger than x_1^*. To show this, we shall define D as a set of states of nature in which $e<1$ and $\sim D$ as the compliment of D. Then we can rewrite (9) as

$$-\int_D MU_2(e)H(e)f(e)de=\int_{-D} MU_2(e)H(e)f(e)de \tag{14}$$

If $H(e)$ is a decreasing function, then

$$\max_{\sim D} H(e)<\min_D H(e) \tag{15}$$

Considering (14) and (15) gives

$$-\int_D MU_2(e)f(e)de<\int_{\sim D} MU_2(e)f(e)de \tag{16}$$

Therefore,

$$0<E\left[\left\{\frac{-(1+r)}{e}+(1+r^*)\right\}U'\left\{\frac{(1+r)(W-x_1^*)}{e}\right\}+(1+r^*)x_1^*\right] \tag{17}$$

which immediately implies that $x_1^*<x_2^*$.

$H'(e)<0$ means that marginal domestic currency returns do not increase as much as marginal foreign currency returns when the foreign currency appreciates.

4. The role of risk aversion: A CRRA utility example

To see the role of risk aversion in portfolio behavior, it is helpful to specify the utility function as belonging to the constant relative risk aversion class (CRRA).

$$U(y)=\begin{cases}\dfrac{y^{1-\frac{1}{\sigma}}}{1-\dfrac{1}{\sigma}} & (0<\sigma)\\[2ex] \log y & (\sigma=1)\end{cases} \tag{18}$$

Consider the corner solutions first. Under the CRRA class of utility functions, (5) is reduced to (8)

$$If\ x_1^* = W, \qquad then\ E\left[\frac{1}{e}\right] \leq 1 - \frac{\mathrm{cov}\left[e^{1-\frac{1}{\sigma}}, \frac{1}{e}\right]}{E\left[e^{1-\frac{1}{\sigma}}\right]} \qquad \sigma \neq 1 \tag{19}$$

$$E\left[\frac{1}{e}\right] \leq \frac{1+r^*}{1+r} \qquad \sigma = 1 \tag{20}$$

As long as $\sigma<1$, the covariance is nonnegative. In this case, $x_1^*=W$. This means $E[1/e] \leq (1+r^*)/(1+r)$, hence $x_2^*=W$.

Similarly, (6) is reduced to

$$If\ x_2^* = 0, \qquad then\ E[e] + \frac{\mathrm{cov}\left[e, \left(\frac{1}{e}\right)^{1-1/\sigma}\right]}{E\left[\left(\frac{1}{e}\right)^{1-1/\sigma}\right]} \leq \frac{1+r}{1+r^*} \qquad \sigma \neq 1 \tag{21}$$

$$E[e] \leq \frac{1+r}{1+r^*} \qquad \sigma = 1 \tag{22}$$

Therefore, x_1^* is always 0 when $x_2^*=0$ as long as $\sigma \leq 1$.

Additionally, we find that as long as $\sigma \leq 1$, the inner solution x_2^* is not smaller than x_1^*. Since $H'(e)=(1-1/\sigma)e^{-1/\sigma}(\sigma \neq 1)$, $H'(e)$ is negative when $1<\sigma$. As we have seen, this is a sufficient condition for x_2^* to be larger than x_1^*. With log utility, $H'(e)$ is equal to zero, which implies that $x_1^*=x_2^*$. Therefore, we can rule out the possibility that $x_1^*>x_2^*$ under the CRRA class of utility functions with $\sigma \leq 1$.

5. Conclusion

In this paper, we analyzed the effect of a change of currency denomination on portfolio choice. Despite expectations of exchange rate fluctuation, foreign currency asset holdings can be a larger proportion of an agent's

(8) See Appendix (C).

portfolio if returns are measured in domestic currency rather than in terms of foreign currency. However, when the degree of risk aversion is high enough, such a situation will never occur. In addition, currency denomination does not affect portfolio behavior under a log utility function.

Appendix

(A) The Kuhn-Tucker conditions

There exist λ_1 and $\lambda_2 \geq 0$ such that

$$\lambda_1 x_1^* = 0 \tag{A1}$$

$$\lambda_2 (W - x_1^*) = 0 \tag{A2}$$

$$E[\{-(1+r)+(1+r^*)e\}U'\{(1+r)(W-x_1^*)+(1+r^*)ex_1^*\}] + \lambda_1 - \lambda_2 = 0 \tag{A3}$$

If $x_1^* = 0$, then $\lambda_2 = 0$ from (A2). Therefore (A3) is

$$\lambda_1 = -U'\{(1+r)W\}E[-(1+r)+(1+r^*)e].$$

Since $\lambda_1 \geqq 0$ and $U' > 0$,

$$E[-(1+r)+(1+r^*)e] \leq 0$$

$$E[e] \leq \frac{1+r}{1+r^*}.$$

If $0 < x_1^* < W$, then $\lambda_1 = \lambda_2 = 0$ from (A1) and (A2). Therefore (A3) is

$$E[\{(1+r)+(1+r^*)e\}U'\{(1+r)(W-x_1^*)+(1+r^*)ex_1^*\}] = 0 \tag{A4}$$

(A4) is rewritten as follows:

$$E[-(1+r)+(1+r^*)e]E[U'\{(1+r)(W-x_1^*)+(1+r^*)ex_1^*\}]$$
$$+\operatorname{cov}[-(1+r)+(1+r^*)e, U'\{(1+r)(W-x_1^*)\}+(1+r^*)ex_1^*] = 0$$

Since the covariance is negative and $E[U'\{\cdot\}]$ is positive,

$$E[-(1+r)+(1+r^*)e] > 0$$

$$\frac{1+r}{1+r^*} < E[e]$$

Similarly, we can obtain the Kuhn-Tucker conditions for problem (2).

(B) Expanding $1/e$ around $1/E(e)$ by Taylor's series and taking expected values will yield the following approximation. (See Siegel (1972)).

$$E\left[\frac{1}{e}\right]=\frac{1+\dfrac{VAR[e]}{(E[e])^2}}{E[e]}$$

Therefore, $(1+r)/(1+r^*)<E[1/e]$ implies

$$\{E[e]\}^2\left\{\frac{1+r}{1+r^*}E[e]-1\right\}<VAR[e].$$

When the variance is large enough, $(1+r)/(1+r^*)<E[e]$ can hold simultaneously.

(C) When x_1^* is W, (5) is rewritten as

$$\frac{1+r}{1+r^*}-\mathrm{cov}\left[e,\frac{U'\{(1+r^*)eW\}}{E[U'\{(1+r^*)eW\}]}\right]\leq E[e].$$

In the case of $\sigma\neq1$,

$$\frac{1+r}{1+r^*}-\mathrm{cov}\left[e,\frac{e^{-1/\sigma}}{E[e^{-1/\sigma}]}\right]\leq E[e]$$

$$\frac{1+r}{1+r^*}+E[e]E\left[\frac{e^{-1/\sigma}}{E[e^{-1/\sigma}]}\right]-E\left[e\frac{e^{-1/\sigma}}{E[e^{-1/\sigma}]}\right]\leq E[e]$$

$$\frac{1+r}{1+r^*}E\left[\frac{1}{e}e^{1-1/\sigma}\right]\leq E[e^{1-1/\sigma}]$$

$$\frac{1+r}{1+r^*}\left\{E\left[\frac{1}{e}\right]E[e^{1-1/\sigma}]+\mathrm{cov}\left[\frac{1}{e},e^{1-1/\sigma}\right]\right\}\leq E[e^{1-1/\sigma}].$$

By arranging the above inequality, we can get (19).

In the case of $\sigma=1$,

$$\frac{1+r}{1+r^*}-\mathrm{cov}\left[e,\frac{\dfrac{1}{e}}{E\left[\dfrac{1}{e}\right]}\right]\leq E[e]$$

$$\frac{1+r}{1+r^*}+E[e]E\left[\frac{\dfrac{1}{e}}{E\left[\dfrac{1}{e}\right]}\right]-E\left[e\frac{\dfrac{1}{e}}{E\left[\dfrac{1}{e}\right]}\right]\leq E[e]$$

$$\frac{1+r}{1+r^*}\leq\frac{1}{E\left[\dfrac{1}{e}\right]}.$$

By re-arranging the above inequality, we get (20). Similarly, we can get (21) and (22).

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